

CHAPTER 4

MARKET AND DEMAND ANALYSIS

1. The sales of a certain product during a 16- year period have been as follows.

Period	Sales	Period	Sales
1	560	9	710
2	580	10	700
3	620	11	730
4	600	12	760
5	630	13	750
6	660	14	780
7	640	15	820
8	680	16	810

Find the least squares regression line for the above data.

Solution:

We have to estimate the parameters a and b in the linear relationship

$$Y_t = a + bT \text{ using the least squares method.}$$

According to the least squares method the parameters are:

$$b = \frac{\sum TY - n \bar{T} \bar{Y}}{\sum T^2 - n \bar{T}^2}$$

$$a = \bar{Y} - b\bar{T}$$

The parameters are calculated below:

T	Y	TY	T^2
1	560	560	1
2	580	1160	4
3	620	1860	9
4	600	2400	16
5	630	3150	25
6	660	3960	36
7	640	4480	49
8	680	5440	64
9	710	6390	81
10	700	7000	100
11	730	8030	121
12	760	9120	144
13	750	9750	169

14	780	10920	196
15	820	12300	225
16	810	12960	256

$$\begin{array}{llll} \Sigma T=136 & \Sigma Y=11,030 & \Sigma TY=99,480 & \Sigma T^2=1,496 \\ \bar{T}=8.5 & \bar{Y}=689.4 & & \end{array}$$

$$b = \frac{\Sigma TY - n \bar{T} \bar{Y}}{\Sigma T^2 - n \bar{T}^2} = \frac{99,480 - 16 \times 8.5 \times 689.4}{1,496 - 16 \times 8.5 \times 8.5}$$

$$= \frac{5,721.6}{340} = 16.8$$

$$\begin{aligned} a &= \bar{Y} - b \bar{T} \\ &= 689.4 - 16.8 (8.5) \\ &= 546.6 \end{aligned}$$

Thus linear regression is
 $Y = 546.6 + 16.8 T$

2. For the data given in Problem 1 assume that the forecast for period 1 was 550. If α is equal to 0.2, derive the forecasts for the periods 2 to 16 using the exponential smoothing method.

Solution:

In general, in exponential smoothing the forecast for $t + 1$ is

$$F_{t+1} = F_t + \alpha e_t$$

F_1 is given to be 550 and α is given to be 0.2

The forecasts for periods 2 to 16 are calculated below:

Period t	Data (S_t)	Forecast (F_t)	Error $e_t = (S_t - F_t)$	Forecast for $t + 1$ ($F_{t+1} = F_t + \alpha e_t$)
1	560	550.0	10.0	$F_2 = 550 + 0.2 \times 10 = 552.0$
2	580	552.0	28.0	$F_3 = 552 + 0.2 \times 28 = 557.6$
3	620	557.6	62.4	$F_4 = 557.6 + 0.2 \times 62.4 = 570.1$
4	600	570.1	29.9	$F_5 = 570.1 + 0.2 \times 29.9 = 576.1$
5	630	576.1	53.9	$F_6 = 576.1 + 0.2 \times 53.9 = 586.9$
6	660	586.9	73.1	$F_7 = 586.9 + 0.2 \times 73.1 = 601.5$
7	640	601.5	38.5	$F_8 = 601.5 + 0.2 \times 38.5 = 609.2$
8	680	609.2	70.8	$F_9 = 609.2 + 0.2 \times 70.8 = 623.3$
9	710	623.3	86.7	$F_{10} = 623.3 + 0.2 \times 86.7 = 640.7$
10	700	640.7	59.3	$F_{11} = 640.7 + 0.2 \times 59.3 = 652.5$
11	730	652.5	77.5	$F_{12} = 652.5 + 0.2 \times 77.5 = 668.0$

12	760	668.0	92.0	$F_{13} = 668.0 + 0.2 \times 92 = 686.4$
13	750	686.4	63.6	$F_{14} = 686.4 + 0.2 \times 63.6 = 699.1$
14	780	699.1	80.9	$F_{15} = 699.1 + 0.2 \times 80.9 = 715.3$
15	820	715.3	104.7	$F_{16} = 715.3 + 0.2 \times 104.7 = 736.3$
16	810	736.3	73.7	

3. For the data given in problem 1, set $n = 4$ and develop forecasts for the periods 5 to 16 using the moving average method.

Solution:

According to the moving average method

$$F_{t+1} = \frac{S_t + S_{t-1} + \dots + S_{t-n+1}}{n}$$

where F_{t+1} = forecast for the next period

S_t = sales for the current period

n = period over which averaging is done

Given $n = 4$, the forecasts for the period 5 to 16 are given below:

Period t	Data (S_t)	Forecast (F_t)	Forecast for $t + 1$ $F_{t+1} = (S_t + S_{t-1} + S_{t-2} + S_{t-3}) / 4$
1	560		
2	580		
3	620		
4	600		$F_5 = (560 + 580 + 620 + 600) / 4 = 590$
5	630	590.0	$F_6 = (580 + 620 + 600 + 630) / 4 = 607.5$
6	660	607.5	$F_7 = (620 + 600 + 630 + 660) / 4 = 627.5$
7	640	627.5	$F_8 = (600 + 630 + 660 + 640) / 4 = 632.5$
8	680	632.5	$F_9 = (630 + 660 + 640 + 680) / 4 = 652.5$
9	710	652.5	$F_{10} = (660 + 640 + 680 + 710) / 4 = 672.5$
10	700	672.5	$F_{11} = (640 + 680 + 710 + 700) / 4 = 682.5$
11	730	682.5	$F_{12} = (680 + 710 + 700 + 730) / 4 = 705.0$
12	760	705.0	$F_{13} = (710 + 700 + 730 + 760) / 4 = 725.0$
13	750	725.0	$F_{14} = (700 + 730 + 760 + 750) / 4 = 735.0$
14	780	735.0	$F_{15} = (730 + 760 + 750 + 780) / 4 = 755.0$
15	820	755.0	$F_{16} = (760 + 750 + 780 + 820) / 4 = 777.5$
16	810	777.5	

4. The following information is available on quantity demanded and income level:

Q_1 = Quantity demanded in the base year = 200

Q_2 = Quantity demanded in the following year = 250

I_1 = Income level in base year = 400

I_2 = Income level in the following year = 600

What is the income elasticity of demand?

$$\text{Income Elasticity of Demand} = \frac{Q_2 - Q_1}{I_2 - I_1} \times \frac{I_1 + I_2}{Q_1 + Q_2}$$

$$= \frac{250 - 200}{600 - 400} \times \frac{400 + 600}{200 + 250}$$

$$= 0.56$$

5. The following information is available on price and quantity for a certain product:

Price per unit demanded in the base year (P_1) = Rs.20

Quantity demanded in the base year (Q_1) = 50,000

Price per unit demanded in the following year (P_2) = Rs.30

Quantity demanded in the following year (Q_2) = 45,000

$$\text{Price Elasticity of Demand} = \frac{Q_2 - Q_1}{P_2 - P_1} \times \frac{P_1 + P_2}{Q_2 + Q_1}$$

$$= \frac{45,000 - 50,000}{30 - 20} \times \frac{20 + 30}{45,000 + 50,000} = -0.26$$

CHAPTER 6

FINANCIAL ESTIMATES AND PROJECTIONS

1. The balance sheet of Sushil Corporation at the end of year n (the year which is just over) is as follows:

(Rs in million)			
Liabilities		Assets	
Share capital	50	Fixed assets	110
Reserves and surplus	20	Investments	6
Secured loans	30	Current assets	26
Unsecured loans	25	• Cash	4
Current liabilities	12	• Receivables	12
Provisions	5	• Inventories	10
	142		142

The projected income statement and the distribution of earnings is given below:

(Rs in million)	
Sales	250
Cost of goods sold	160
Depreciation	20
Profit before interest and taxes	70
Interest	10
Profit before tax	60
Tax	18
Profit after tax	42
Dividends	10
Retained earnings	32

During the year $n+1$, the firm plans to raise a secured term loan of Rs 10 million, repay a previous secured term loan to the extent of Rs 18 million. Current liabilities and provisions would increase by 10 per cent. Further, the firm plans to acquire fixed assets worth Rs 40 million and raise its inventories by Rs 2 million. Receivables are expected to increase by 8 per cent. The level of cash would be the balancing amount in the projected balance sheet.

Given the above information, prepare the following:

- (i) Projected cash flow statement
- (ii) Projected balance sheet

Solution:

Projected Cash Flow Statement (Rs. in million)

Sources of Funds

Profit before interest and tax	70
Depreciation provision for the year	20
Secured term loan	<u>10</u>
Total (A)	<u>100</u>

Disposition of Funds

Capital expenditure	40
Increase in working capital*	1.26
Repayment of term loan	18.0
Interest	10
Tax	18
Dividends	<u>10</u>
Total (B)	<u>97.26</u>

Opening cash balance	4.00
Net surplus (deficit) (A – B)	2.74
Closing cash balance	6.74

Projected Balance Sheet

(Rs. in million)

Liabilities		Assets	
Share capital	50	Fixed assets	130
Reserves & surplus	52	Investments	6
Secured loans	22	Current assets	
Unsecured loans	25	* Cash	6.74
Current liabilities		* Receivables	12.96
& provisions	<u>18.70</u>	* Inventories	<u>12.00</u>
	167.70		167.70

* Working capital here is defined as :

(Current assets other than cash) – (Current liabilities other than bank borrowings)

In this case inventories increase by 2 million, receivables increase by 0.96 million and current liabilities and provisions increase by 1.7 million. So working capital increases by 1.26 million

2. Modern Electronics Limited is being set up to manufacture electronic components. The expected outlays and proposed financing during the construction and the first operating year are shown below:

	<i>Construction period</i>	<i>1 Operating Year</i>
Outlays		
Land	30	-
Buildings	100	-
Plant & machinery	500	-
Miscellaneous fixed assets	105	-
Preliminary expenses	25	-
Pre-operative expenses	100	-
Current assets (other than cash)		<u>480</u>
	<u>860</u>	<u>480</u>
Financing		
Equity capital	360	-
Term loan	540	120
Short-term bank borrowing		<u>360</u>
	<u>900</u>	<u>480</u>

The following information is available:

- The construction period will last for one year, beginning on 1st April of year n and ending on 31st March of year $n+1$.
- The first operating period will begin on 1st April of year $n+1$ and end on 31st March of year $n+2$.
- The term loan will carry an interest of 16 percent. It is repayable in 16 equal semi-annual instalments, the first instalment falling due in the middle of the second operating year. The interest on term loan during the construction period is included in pre-operative expenses. The term loan financing of 120 in the first operating period will occur right in the beginning of that year.
- Short-term bank borrowing of 360 will occur right in the beginning of the first operating year. It will carry an interest rate of 18 percent.
- Pre-operative expenses will be allocated to land, building, plant and machinery, and miscellaneous fixed assets in proportion of their values. Preliminary expenses will be written off in ten equal annual instalments.
- The expected revenues and cost of sales (excluding depreciation, other amortisation, and interest) for the first operating year are 900 and 650 respectively.
- The depreciation rates for company law purposes will be as follows :

Building	: 3.34 percent
Plant and machinery	: 10.34 percent

Miscellaneous fixed assets : 10.34 percent

h. There will be no income tax liability for the first operating year.

Given the above information, complete the following projected statements.

Projected Income Statement for the I Operating Year

Sales	900
Cost of sales	650
Depreciation	
Interest	
Write-off of preliminary expenses	2.5
Net profit	

Projected Cash Flow Statement

Sources	Construction period	I Operating year
• Equity capital	360	
• Term loan	540	120
• Short-term bank borrowing	Nil	360
• Profit before interest and taxes		
• Depreciation		
• Writeoff of preliminary expenses		2.5
Total	900	
Uses		
• Capital expenditure	735	Nil
• Current assets (other than cash)	Nil	480
• Preliminary expenses	25	Nil
• Preoperative expenses	100	Nil
• Interest	Nil	
	860	
• Opening cash balance	0	40
• Net surplus/deficit	40	
• Closing balance	40	

Projected Balance Sheet

<i>Liabilities</i>	<i>31/3/n+1</i>	<i>31/3/n+2</i>	<i>Assets</i>	<i>31/3/n+1</i>	<i>31/3/n+2</i>
•Share capital	360	360	Fixed assets (net)	835	
•Reserves & surplus	Nil				
•Secured loans			Current assets:		
• Term loan	540	660	• Cash	40	
• Short-term bank borrowing	Nil	360	• Other current assets	Nil	
•Unsecured loans	Nil	Nil			
•Current liabilities and provisions	Nil	Nil	Miscellaneous expenditures & losses		
			• Preliminary expenses	25	22.5
	<u>900</u>	<u>.....</u>		<u>900</u>	<u>.....</u>

Working:

Depreciation

<i>Basic cost</i>	<i>Allocation of Preoperative exp</i>	<i>Asset value</i>	<i>Depr'n Rate</i>	<i>Depr'n</i>
Land : 30	$30/735 \times 100 = 4.08$	34.08	-	-
Building : 100	$100/735 \times 100 = 13.61$	113.61	3.34%	3.79
Plant & mach: 500	$500/735 \times 100 = 68.03$	568.03	10.34%	58.73
MFA : 105	$105/735 \times 100 = 14.29$	119.29	10.34%	12.33
735				<u>74.85</u>

Interest

Interest on term loan : 16% on 660 = 105.6
Interest on short-term bank borrowing: 18% on 360 = 64.8
170.4

Projected Income Statement for the I Operating Year

Sales	900
Cost of sales	650
Depreciation	74.85
Interest	170.4
Writeoff of preliminary expenses	2.5
Net profit	2.25

Projected Cash Flow Statement

Sources	Construction period I Operating year	
• Equity capital	360	
• Term loan	540	120
• Short-term bank borrowing	Nil	360
• Profit before interest and taxes		172.65
• Depreciation		74.85
• Writeoff of preliminary expenses		2.5
Total	900	730
Uses		
• Capital expenditure	735	Nil
• Current assets (other than cash)	Nil	480
• Preliminary expenses	25	Nil
• Preoperative expenses	100	Nil
• Interest	Nil	170.4
	860	650.4
• Opening cash balance	0	40
• Net surplus/deficit	40	79.6
• Closing balance	40	119.6

Projected Balance Sheet

<i>Liabilities</i>	<i>31/3/n+1</i>	<i>31/3/n+2</i>	<i>Assets</i>	<i>31/3/n+1</i>	<i>31/3/n+2</i>
•Share capital	360	360	Fixed assets (net)	835	760.15
•Reserves & surplus	Nil	2.25			
•Secured loans			Current assets:		
• Term loan	540	660	• Cash	40	119.6
• Short-term bank borrowing	Nil	360	• Other current assets	Nil	480
•Unsecured loans	Nil	Nil			
•Current liabilities and provisions	Nil	Nil	Miscellaneous expenditures & losses		
			• Preliminary expenses	25	22.5
	900	1382.25		900	1382.25

CHAPTER 7

THE TIME VALUE OF MONEY

1. Calculate the value 10 years hence of a deposit of Rs 5,000 made today if the interest rate is (a) 7 percent, (b) 9 percent, (c) 11 percent, and (d) 14 percent.

Solution:

Value five years hence of a deposit of Rs.1,000 at various interest rates is as follows:

$$\begin{array}{llll} r & = & 7\% & FV_{10} = 5000 \times FVIF (7\%, 10 \text{ years}) \\ & & & = 5000 \times 1.967 = \text{Rs.9835} \end{array}$$

$$\begin{array}{llll} r & = & 9\% & FV_{10} = 5000 \times FVIF (9\%, 10 \text{ years}) \\ & & & = 5000 \times 2.367 = \text{Rs.11835} \end{array}$$

$$\begin{array}{llll} r & = & 11\% & FV_{10} = 5000 \times FVIF (11\%, 10 \text{ years}) \\ & & & = 5000 \times 2.839 = \text{Rs.14195} \end{array}$$

$$\begin{array}{llll} r & = & 14\% & FV_{10} = 5000 \times FVIF (14\%, 10 \text{ years}) \\ & & & = 5000 \times 3.707 = \text{Rs.18535} \end{array}$$

2. If you deposit Rs 2,000 today at 9 percent rate of interest in how many years (roughly) will this amount grow to Rs 32,000? Work this problem using the *rule of 72* — do not use tables.

Solution:

$$\text{Rs.32,000} / \text{Rs. 2,000} = 16 = 2^4$$

According to the Rule of 72 , at 9 percent, interest rate doubling takes place approximately in $72 / 9 = 8$ years

So Rs.2000 will grow to Rs.32,000 in approximately 4×8 years = 32years

3. A finance company offers to give Rs 12,000 after 16 years in return for Rs 3,000 deposited today. Using the *rule of 69*, figure out the approximate interest offered.

Solution:

In 16 years Rs.3000 grows to Rs.12,000 or 4 times. This is 2^2 times the initial deposit. Hence doubling takes place in $16 / 2 = 8$ years.

According to the Rule of 69, the doubling period is:

$$0.35 + 69 / \text{Interest rate}$$

Equating this to 8 and solving for interest rate, we get

$$\text{Interest rate} = 9.02 \%$$

4. Nitin can save Rs 5,000 a year for 4 years, and Rs 6,000 a year for 6 years thereafter. What will these savings cumulate to at the end of 10 years, if the rate of interest is 8 percent?

Solution:

Saving Rs.5000 a year for 4 years and Rs.6000 a year for 6 years thereafter is equivalent to saving Rs.5000 a year for 10 years and Rs.1000 a year for the years 5 through 10.

Hence the savings will cumulate to:

$$\begin{aligned} & 5000 \times \text{FVIFA} (8 \%, 10 \text{ years}) + 1000 \times \text{FVIFA} (8\%, 6 \text{ years}) \\ & = 5000 \times 14.487 + 1000 \times 7.336 = \text{Rs.}79,771. \end{aligned}$$

5. Santosh plans to purchase an apartment costing Rs.10,000,000 after 5 years. How much should he save annually to have a sum of Rs 10,000,000 at the end of 5 years, if the interest rate is 9 percent?

Solution:

Let A be the annual savings.

$$\begin{aligned} A \times \text{FVIFA} (9\%, 5 \text{ years}) & = 10,000,000 \\ A \times 5.985 & = 10,000,000 \end{aligned}$$

$$\text{So } A = 10,000,000 / 5.985 = \text{Rs.}1,670,844$$

6. A finance company promises to pay a Rs 100,000 at the end of 4 years to investors who deposit Rs.18,182 at the end of each year for the next four years. What interest rate is implicit in this offer?

Solution:

$$18,182 \times \text{FVIFA} (r, 4 \text{ years}) = 100,000$$

$$\text{FVIFA} (r, 4 \text{ years}) = 100,000 / 18,182 = 5.5$$

From the tables we find that

$$\begin{aligned}\text{FVIFA (20\%, 4 years)} &= 5.368 \\ \text{FVIFA (24\%, 4 years)} &= 5.684\end{aligned}$$

Using linear interpolation in the interval, we get:

$$r = 20\% + \frac{(5.5 - 5.368)}{(5.684 - 5.368)} \times 4\% = 21.67\%$$

7. A promises to give B Rs 25,000 after 6 years in exchange for Rs 13,000 today. What interest rate is implicit in this offer?

Solution:

$$\begin{aligned}13,000 \times \text{FVIF}(r, 6 \text{ years}) &= 25,000 \\ \text{FVIF}(r, 6 \text{ years}) &= 25,000 / 13,000 = 1.923\end{aligned}$$

From the tables we find that

$$\begin{aligned}\text{FVIF (11\%, 6 years)} &= 1.870 \\ \text{FVIF (12\%, 6 years)} &= 1.974\end{aligned}$$

Using linear interpolation in the interval, we get:

$$r = 11\% + \frac{(1.923 - 1.870) \times 1\%}{(1.974 - 1.870)} = 11.51\%$$

8. Find the present value of Rs 50,000 receivable after 5 years if the rate of discount is (i) 16 percent, (ii) 10 percent.

Solution:

$$\begin{aligned}\text{(i)} \quad \text{PV} &= 50,000 \text{ PVIF (16\%, 5 years)} = 50,000 \times 0.476 = \text{Rs.}23,800 \\ \text{(ii)} \quad \text{PV} &= 50,000 \text{ PVIF (10\%, 5 years)} = 50,000 \times 0.621 = \text{Rs.}31,050\end{aligned}$$

9. What is the present value of a 15-year ordinary annuity of Rs 5,000 at 8 percent?

Solution:

$$\text{PV} = 5,000 \times \text{PVIFA (8\%, 15 years)} = 5,000 \times 8.559 = \text{Rs.}42,795$$

10. At the time of his retirement, Mr.Kamat is given a choice between two alternatives: (a) an annual pension of Rs 200,000 as long as he lives, and (b) a lump sum amount of Rs 1,500,000. If Mr.Kamat expects to live for 20 years and the interest rate is 10 percent, which option appears more attractive?

Solution:

The present value of an annual pension of Rs.200,000 for 20 years when $r = 10\%$ is: $200,000 \times \text{PVIFA}(10\%, 20 \text{ years}) = 200,000 \times 8.514 = \text{Rs.}1,702,800$.

As this amount is greater than the lumpsum offer of Rs.1,500,000, he should go for the pension option.

11. If you deposit Rs 800,000 in a bank which pays 8 percent interest how much can you withdraw at the end of each year for a period of 10 years. Assume that at the end of 10 years the amount deposited will whittle down to zero.

Solution:

The amount that can be withdrawn annually is:

$$A = \frac{800,000}{\text{PVIFA}(8\%, 10 \text{ years})} = \frac{800,000}{6.710} = \text{Rs.}119,225$$

12. What is the present value of an income stream which provides Rs 3,000 at the end of year one, Rs 4,500 at the end of year two, and Rs 7,000 at the end of each of the years 3 through 8, if the discount rate is 15 percent?

Solution:

The present value of the income stream is:

$$\begin{aligned} & 3,000 \times \text{PVIF}(15\%, 1 \text{ year}) + 4,500 \times \text{PVIF}(15\%, 2 \text{ years}) \\ & + 7,000 \times \text{PVIFA}(15\%, 6 \text{ years}) \times \text{PVIF}(15\%, 2 \text{ years}) \\ & = 3,000 \times 0.870 + 4,500 \times 0.756 + 7,000 \times 3.784 \times 0.756 = \text{Rs.}26,037. \end{aligned}$$

13. What is the present value of an income stream which provides Rs 10,000 a year for the first four years and Rs 15,000 a year forever thereafter, if the discount rate is 9 percent?

Solution:

The present value of the income stream is:

$$\begin{aligned} & 10,000 \times \text{PVIFA} (9 \%, 4 \text{ years}) + 15,000/0.09 \times \text{PVIF} (9 \%, 4 \text{ years}) \\ & = 10,000 \times 3.240 + 15,000/0.09 \times 0.708 \\ & = \text{Rs.}150,400 \end{aligned}$$

14. What amount must be deposited today in order to earn an annual income of Rs 20,000 beginning from the end of 10 years from now? The deposit earns 8 percent per year.

Solution:

To earn an annual income of Rs.20,000 beginning from the end of 10 years from now, if the deposit earns 8 % per year, a sum of

$$\text{Rs.}20,000 / 0.08 = \text{Rs.}250,000$$

is required at the end of 9 years. The amount that must be deposited to get this sum is:

$$\text{Rs.}250,000 \text{ PVIF} (8 \%, 9 \text{ years}) = \text{Rs.}250,000 \times 0.5 = \text{Rs.}125,000$$

15. Investment Trust offers you the following financial contract. If you deposit Rs 10 000 with them they promise to pay Rs 2,500 annually for 8 years. What interest rate would you earn on this deposit?

Solution:

$$\text{Rs.}10,000 = \text{Rs.}2,500 \times \text{PVIFA} (r, 8 \text{ years})$$

$$\text{PVIFA} (r, 8 \text{ years}) = \text{Rs.}10,000 / \text{Rs.}2,500 = 4$$

From the tables we find that:

$$\text{PVIFA} (18\%, 8 \text{ years}) = 4.078$$

$$\text{PVIFA} (19\%, 8 \text{ years}) = 3.954$$

Using linear interpolation we get:

$$\begin{aligned} r &= 18 \% + \left[\frac{4.078 - 4.00}{4.078 - 3.954} \right] \times 1 \% \\ &= 18.63 \% \end{aligned}$$

16. Suppose you deposit Rs 50,000 with an investment company which pays 12 percent interest with quarterly compounding. How much will this deposit grow to in 6 years?

Solution:

$$\begin{aligned}
 FV_6 &= \text{Rs.}50,000 [1 + (0.12 / 4)]^{6 \times 4} \\
 &= \text{Rs.}50,000 (1.03)^{24} \\
 &= \text{Rs.}50,000 \times 2.033 \\
 &= \text{Rs.}101,640
 \end{aligned}$$

17. How much would a deposit of Rs 10,000 at the end of 10 years be, if the interest rate is 8 percent and if the compounding is done once in six months?

Solution:

$$\begin{aligned}
 FV_{10} &= \text{Rs.}10,000 [1 + (0.08/2)]^{10 \times 2} \\
 &= \text{Rs.}10,000 (1.04)^{20} \\
 &= \text{Rs.}10,000 \times 2.191 \\
 &= \text{Rs.}21,911
 \end{aligned}$$

18. What is the difference between the effective rate of interest and stated rate of interest in the following cases:
Case I: Stated rate of interest is 10 percent and the frequency of compounding is four times a year.
Case II: Stated rate of interest is 16 percent and the frequency of compounding is three times a year.

Solution:

	<i>I</i>	<i>II</i>
Stated rate (%)	10	16
Frequency of compounding	4 times	3 times
Effective rate (%)	$(1 + 0.10/4)^4 - 1$ = 10.38	$(1 + 0.16/3)^3 - 1$ = 16.87
Difference between the effective rate and stated rate (%)	0.38	0.87

19. If the interest rate is 10 percent how much investment is required now to yield an income of Rs 15,000 per year from the beginning of the 6th year and which continues thereafter forever?

Solution:

Investment required at the end of 4th year to yield an income of Rs.15,000 per year from the end of 5th year (beginning of 6th year) for ever:

$$\begin{aligned} & \text{Rs.15,000} \times \text{PVIFA}(10\%, \infty) \\ &= \text{Rs.15,000} / 0.10 = \text{Rs.150,000} \end{aligned}$$

To have a sum of Rs.150,000 at the end of 4th year, the amount to be deposited now is:

$$\frac{\text{Rs.150,000}}{0.683} \times \text{PVIF}(10\%, 4 \text{ years}) = \text{Rs.150,000} \times 0.683 = \text{Rs.102,450}$$

20. You have a choice between Rs 10,000 now and Rs 25,000 after 10 years. Which would you choose? What does your preference indicate?

Solution:

Rs.25,000 receivable after 10 years is equivalent to receiving Rs. $25,000 / (1+r)^{10}$ now. I would be indifferent if $25,000 / (1+r)^{10} = 10,000$ or $(1+r)^{10} = 2.5$ in which case $r = 9.6$ percent. If I choose to receive Rs.25,000 after 10 years it indicates that I consider 9.6 percent rate of interest to be a good one in the prevailing market conditions. If I choose to receive Rs.10,000 now it indicates that I do not consider 9.6 percent rate of interest attractive in the prevailing market conditions.

21. A deposits Rs 50,000 in a bank for five years under its cumulative deposit scheme. The interest rate is 8 percent and compounding is done quarterly. What will be the maturity value of the deposit? If the inflation rate is 4 percent per year, what will be the value of the deposit after 5 years in terms of the current rupees?

Solution:

$$\begin{aligned} \text{FV}_5 &= \text{Rs.50,000} [1 + (0.08 / 4)]^{5 \times 4} \\ &= \text{Rs.50,000} (1.02)^{20} \\ &= \text{Rs.50,000} \times 1.486 \\ &= \text{Rs.74,300} \end{aligned}$$

If the inflation rate is 4 % per year, the value of Rs.74,300 5 years from now, in terms of the current rupees is:

$$\begin{aligned} & \text{Rs. 74,300} \times \text{PVIF} (4\%, 5 \text{ years}) \\ &= \text{Rs. 74,300} \times 0.822 = \text{Rs.61,075} \end{aligned}$$

22. How much should be deposited at the beginning of each year for 5 years in order to provide a sum of Rs 1,000,000 at the end of 5 years if the interest rate is 8 percent?

Solution:

A constant deposit at the beginning of each year represents an annuity due.

PVIFA of an annuity due is equal to : PVIFA of an ordinary annuity $\times (1 + r)$

To provide a sum of Rs.1,000,000 at the end of 5 years the annual deposit should be

$$\begin{aligned} A &= \frac{\text{Rs.1,000,000}}{\text{FVIFA}(8\%, 5 \text{ years}) \times (1.08)} \\ &= \frac{\text{Rs.1,000,000}}{5.867 \times 1.08} = \text{Rs.157,819} \end{aligned}$$

23. Suresh requires Rs 100,000 at the beginning of each year from 2020 to 2024. How much should he deposit at the end of each year from 2010 to 2014? The interest rate is 10 percent.

Solution:

The discounted value of Rs.100,000 required at the beginning of each year from 2020 to 2025, evaluated as at the beginning of 2019 (or end of 2018) is:

$$\begin{aligned} &\text{Rs.100,000} \times \text{PVIFA} (10 \%, 4 \text{ years}) \\ &= \text{Rs.100,000} \times 3.170 = \text{Rs.317,000}. \end{aligned}$$

The discounted value of Rs. 317,000 evaluated at the end of 2014 is

$$\begin{aligned} &\text{Rs. 317,000} \times \text{PVIF} (10 \%, 4 \text{ years}) \\ &= \text{Rs. 317,000} \times 0.683 = \text{Rs.216,511} \end{aligned}$$

If A is the amount deposited at the end of each year from 2010 to 2014 then

$$A \times \text{FVIFA} (10 \%, 5 \text{ years}) = \text{Rs. 216,511}$$

$$A \times 6.105 = \text{Rs. 216,511}$$

$$A = \text{Rs. 216,511} / 6.105 = \text{Rs.35,465}$$

24. What is the present value of Rs 10,000 receivable annually for 20 years if the first receipt occurs after 5 years and the discount rate is 8 percent.

Solution:

The discounted value of the annuity of Rs.10,000 receivable for 20 years, evaluated as at the end of 4th year is:

$$\text{Rs.10,000} \times \text{PVIFA} (8 \%, 20 \text{ years}) = \text{Rs.10,000} \times 9.818 = \text{Rs.98,180}$$

The present value of Rs. 98,180 is:

$$\text{Rs. 98,180} \times \text{PVIF} (8 \%, 4 \text{ years})$$

$$= \text{Rs. 98,180} \times 0.735$$

$$= \text{Rs.72,162}$$

25. After 3 years Kumar will receive a pension of Rs 15,000 per month for 20 years. How much can Kumar borrow now at 12 percent interest so that the borrowed amount can be paid with 40 percent of the pension amount? The interest will be accumulated till the first pension amount becomes receivable.

Solution:

40 percent of the pension amount is
 $0.40 \times \text{Rs.}15,000 = \text{Rs. } 6,000$

Assuming that the monthly interest rate corresponding to an annual interest rate of 12% is 1%, the discounted value of an annuity of Rs. 6,000 receivable at the end of each month for 240 months (20 years) is:

$$\begin{aligned} &\text{Rs. } 6,000 \times \text{PVIFA}(1\%, 240) \\ &\text{Rs. } 6,000 \times \frac{(1.01)^{240} - 1}{.01 (1.01)^{240}} = \text{Rs.}544,916 \end{aligned}$$

If Kumar borrows Rs. P today on which the monthly interest rate is 1%

$$\begin{aligned} P \times (1.01)^{36} &= \text{Rs. } 544,916 \\ P &= \frac{\text{Rs.}544,916}{1.431} = \text{Rs.}380,794 \end{aligned}$$

26. Ms.Rita buys a scooter with a bank loan of Rs 50,000. A monthly instalment of Rs 2,000 is payable to the bank for the next 30 months towards the repayment of the loan with interest. What interest rate does the bank charge?

Solution:

$$\text{Rs.}2,000 \times \text{PVIFA}(r, 30 \text{ months}) = \text{Rs.}50,000$$

$$\text{PVIFA}(r, 30) = 50,000 / 2,000 = 25$$

From the tables we find that:

$$\begin{aligned} \text{PVIFA}(1\%, 30) &= 25.808 \\ \text{PVIFA}(2\%, 30) &= 22.397 \end{aligned}$$

Using a linear interpolation

$$r = 1\% + \left[\frac{25.808 - 25.000}{25.808 - 22.397} \right] \times 1\%$$

$$= 1.24\%$$

Thus, the bank charges an interest rate of 1.24 % per month.

The corresponding effective rate of interest per annum is

$$[(1.0124)^{12} - 1] \times 100 = 15.94\%$$

27. Prime Tech Ltd. has to retire Rs 20 million of debentures each at the end of 7, 8, and 9 years from now. How much should the firm deposit in a sinking fund account annually for 3 years, in order to meet the debenture retirement need? The net interest rate earned is 10 percent.

Solution:

The discounted value of the debentures to be redeemed between 7 to 9 years evaluated at the end of the third year is:

$$\begin{aligned} & \text{Rs.20 million} \times \text{PVIF (10 \%, 4 years)} \\ & + \text{Rs.20 million} \times \text{PVIF (10 \%, 5 years)} \\ & + \text{Rs.20 million} \times \text{PVIF (10 \%, 6 years)} \\ & = \text{Rs.20 million} (0.683 + 0.621 + 0.564) \\ & = \text{Rs.37.36 million} \end{aligned}$$

If A is the annual deposit to be made in the sinking fund for the years 1 to 3, then

$$A \times \text{FVIFA (10 \%, 3 years)} = \text{Rs. 37.36 million}$$

$$A \times 3.310 = \text{Rs. 37.36 million}$$

$$A = \text{Rs. 37.36 million} / 3.310 = \text{Rs.11.287 million}$$

28. Mr.Amar receives a provident fund amount of Rs 1,500,000. He deposits it in a bank which pays 9 percent interest. If he withdraws annually Rs 400,000, how long can he do so?

Solution:

Let 'n' be the number of years for which a sum of Rs.400,000 can be withdrawn annually.

$$\text{Rs.400,000} \times \text{PVIFA (9 \%, n)} = \text{Rs.1,500,000}$$

$$\text{PVIFA (9 \%, n)} = \text{Rs.1,500,000} / \text{Rs.400,000} = 3.75$$

From the tables we find that

$$\text{PVIFA (9 \%, 4 years)} = 3.240$$

$$\text{PVIFA (9 \%, 5 years)} = 3.890$$

Thus n is between 4 and 5. Using a linear interpolation we get

$$n = 4 + \left[\frac{3.75 - 3.240}{3.890 - 3.240} \right] \times 1 = 4.78 \text{ years}$$

29. Apex Corporation borrows Rs 10,000,000 at an interest rate of 12 percent. The loan is to be repaid in 5 equal annual instalments payable at the end of each of the next 5 years. Prepare the loan amortisation schedule.

Solution:

$$\begin{aligned} \text{Equated annual installment} &= 10,000,000 / \text{PVIFA}(12\%, 5) \\ &= 10,000,000 / 3.605 \\ &= \text{Rs.}2,773,925 \end{aligned}$$

Loan Amortisation Schedule

Year	Beginning amount	Annual installment	Interest	Principal repaid	Remaining balance
1	10,000,000	2,773,925	1,200,000	1,573,925	8,426,075
2	8,426,075	2,773,925	1,011,129	1,762,796	6,663,279
3	6,663,279	2,773,925	799,593	1,974,332	4,688,947
4	4,688,947	2,773,925	562,674	2,211,251	2,477,696
5	2,477,696	2,773,925	297,324	2,476,601	1,095*

(*) rounding off error

30. Rajesh wants to borrow Rs 3,000,000 to buy a flat. He approaches a housing finance company which charges 12 percent interest. He can pay Rs 500,000 per year toward loan amortisation. What should be the maturity period of the loan?

Solution:

Define n as the maturity period of the loan. The value of n can be obtained from the equation.

$$500,000 \times \text{PVIFA}(12\%, n) = 3,000,000$$

$$\text{PVIFA}(12\%, n) = 6$$

From the tables corresponding to 12 %, we find that :

$$\text{PVIFA}(12\%, 11) = 5.938$$

$$\text{PVIFA}(12\%, 12) = 6.194$$

By linear interpolation, $n = 11 + (6 - 5.938) / (6.194 - 5.938) = 11.24$ years.

31. You are negotiating with the government the right to mine 5,000 tons of manganese ore per year for 20 years. The current price per ton of manganese ore

is Rs 100,000 and it is expected to increase at the rate of 5 percent per year. What is the present value of the manganese ore that you can mine if the discount rate is 15 percent?

Solution:

Expected value of manganese ore mined during year 1 = Rs.525 million

Expected present value of the manganese ore that can be mined over the next 20 years assuming a price escalation of 5 % per annum is

$$= \text{Rs.525 million} \times \left[\frac{1 - (1 + g)^n / (1 + i)^n}{i - g} \right]$$

$$= \text{Rs. 525 million} \times \left[\frac{1 - (1.05)^{15} / (1.15)^{15}}{0.15 - 0.05} \right]$$

$$= \text{Rs. 3908.68 million}$$

CHAPTER 8

INVESTMENT CRITERIA

1. Megatronics Limited is evaluating a project whose expected cash flows are as follows:

<i>Year</i>	<i>Cash flow</i>
0	-500,000
1	100,000
2	200,000
3	300,000
4	100,000

- (i) What is the NPV of the project if the cost of capital is 10 percent?

Solution:

$$\begin{aligned}
 \text{NPV} &= -500,000 + \frac{100,000}{(1.10)} + \frac{200,000}{(1.10)^2} \\
 &+ \frac{300,000}{(1.10)^3} + \frac{100,000}{(1.10)^4} \\
 &= -500,000 + 90,909 + 165,289 + 225,394 + 68,301 \\
 &= 49,893
 \end{aligned}$$

- (ii) What is the IRR of the project?

Solution:

	PVIF @14 %	PV	PVIF @ 15 %	PV
100,000	.877	87,700	.870	87,000
200,000	.769	153,800	.756	151,200
300,000	.675	202,500	.658	197,400
100,000	.529	59,200	.572	57,200
		503,200		492,800
	3200			
14 % + $\frac{3200}{10,400}$		= 14.31 %		

(iii) What is the Modified NPV of the project if the reinvestment rate is 13%?

Solution:

$$\begin{aligned}
 &100,000 (1.443) + 200,000 (1.277) + 300,000 (1.13) \\
 &= 144300 + 255400 + 339000 + 100,000 \\
 &= 838700 \\
 \text{NPV}^* &= \frac{838700}{(1 + \text{COC})^4} - 500,000 \\
 &= 572832 - 500,000 = 72832
 \end{aligned}$$

(iv) What is the Modified IRR (MIRR) of the project if the reinvestment rate is 13%?

Solution: Terminal value of the benefits when the reinvestment rate is 13% is 838,700

$$\text{MIRR} = \left[\frac{838,700}{500,000} \right]^{1/4} - 1 = 13.80 \%$$

(v) What is the unrecovered investment balance at the end of year 2?

Solution:

Period	Unrecovered balance at beg.	Interest @ 14.31 %	Cash flow at the end	Unrecovered balance at the end
1	- 500,000	- 71550	100,000	- 471550
2	- 471550	- 67479	200,000	- 339029

(vi) What is the PI?

Solution:

Depends on the COC
If the COC is 10 %

$$\frac{549893}{500000} = 1.0998$$

2. You are evaluating a project whose expected cash flows are as follows:

<i>Year</i>	<i>Cash flow</i>
0	-1,000,000
1	200,000
2	300,000
3	400,000
4	500,000

What is the NPV of the project (in '000s) if the discount rate is 10 percent for year 1 and rises thereafter by 2 percent every year?

Solution:

$$\begin{aligned}
 PVB &= \frac{200}{(1.10)} + \frac{300}{(1.10)(1.12)} + \frac{400}{(1.10)(1.12)(1.14)} \\
 &+ \frac{500}{(1.10)(1.12)(1.14)(1.16)} \\
 &= 181.82 + 243.51 + 284.80 + 306.90 \\
 &= 1017.03 ; \text{ NPV} = 1017.03 - 1000 = 17.03
 \end{aligned}$$

3. An equipment costs Rs.1,000,000 and lasts for 6 years. What should be the minimum annual cash inflow to justify the purchase of the equipment? Assume that the cost of capital is 12 percent.

Solution:

$$\begin{aligned}
 A \times PVIFA (12\%, 6 \text{ yrs}) &= 1,000,000 \\
 A \times 4.111 &= 1,000,000 \\
 A &= 243,250
 \end{aligned}$$

4. The cash flow stream of a project is given below:

<i>Year</i>	<i>Cash flow</i>
0	-9,000
1	0
2	10,000
3	2,714

What is the unrecovered cash balance at the end of year 2?

Solution:

IRR is the value of r in the equation.

$$9000 = \frac{0}{1+r} + \frac{10,000}{(1+r)^2} + \frac{2,714}{(1+r)^3}$$

r works out to 17 percent

Since the IRR is 17 percent and the cash inflow at the end of year 3 is 2714, the unrecovered investment balance at the end of year 2 will be:

$$\frac{2714}{(1.17)} = 2320$$

5. What is the internal rate of return of an investment which involves a current outlay of Rs. 250,000 and results in an annual cash inflow of Rs. 80,000 for 8 years?

IRR (r) can be calculated by solving the following equations for the value of r .

$$\begin{aligned} 80000 \times \text{PVIFA}(r, 8) &= 250,000 \\ \text{i.e., PVIFA}(r, 8) &= 3.125 \end{aligned}$$

From the tables we find that:

$$\begin{aligned} \text{PVIFA}(24, 8) &= 3.421 \\ \text{PVIFA}(28, 8) &= 3.076 \end{aligned}$$

By linear interpolation, $r = 24 + (3.421 - 3.125) \times 4 / (3.421 - 3.076) = 27.43 \%$

6. What is the internal rate of return of the following cash flow stream?

Year	Cash flow
0	(7,000)
1	10,000
2	(1,000)

The IRR (r) for the given cashflow stream has to be obtained by solving the following equation for the value of r .

$$-7000 + 10000 / (1+r) - 1000 / (1+r)^2 = 0$$

This equation has two roots and therefore the IRR rule breaks down.

7. How much can be paid for a machine which brings in an annual cash inflow of Rs. 50,000 for 8 years? Assume that the discount rate is 15 percent?

Solution:

$$\begin{aligned}\text{The amount that can be paid} &= 50000 \times \text{PVIFA} (15\%, 8) \\ &= 50000 \times 4.487 = \text{Rs. } 224,350\end{aligned}$$

8. A company is considering two mutually exclusive investments, Project X and Project Y. The expected cash flows of these projects are as follows:

Year	Project X	Project Y
0	(5,000)	(2,500)
1	(2,500)	800
2	300	1,000
3	2,000	2,000
4	5,000	2,000
5	6,000	1,500

Which project should it choose if the cost of capital is 15 percent? 45 percent?
When the cost of capital is 15 percent:

$$\begin{aligned}\text{NPV(X)} &= -5,000 - 2,500/(1.15) + 300/(1.15)^2 + 2,000/(1.15)^3 \\ &\quad + 5,000/(1.15)^4 + 6,000/(1.15)^5 \\ &= 209.79 \\ \text{NPV(Y)} &= -2,500 + 800/(1.15) + 1,000/(1.15)^2 + 2,000/(1.15)^3 \\ &\quad + 2,000/(1.15)^4 + 1,500/(1.15)^5 \\ &= 2,156\end{aligned}$$

As the NPV of Project Y is positive and higher than that of Project X, it should choose Project Y.

When the cost of capital is 45 percent:

$$\begin{aligned}\text{NPV(X)} &= -5,000 - 2,500/(1.45) + 300/(1.45)^2 + 2,000/(1.45)^3 \\ &\quad + 5,000/(1.45)^4 + 6,000/(1.45)^5 \\ &= -3,858.25 \\ \text{NPV(Y)} &= -2,500 + 800/(1.45) + 1,000/(1.45)^2 + 2,000/(1.45)^3 \\ &\quad + 2,000/(1.45)^4 + 1,500/(1.45)^5 \\ &= -130.16\end{aligned}$$

As the NPVs are negative it should not choose any of the two projects. ##

9. A company is considering two mutually exclusive projects, P and Q. Project P involves an outlay of Rs.500 million which will generate an expected cash inflow of Rs. 100 million per year for 7 years. Project Q calls for an outlay of Rs. 200 million which will produce an expected cash inflow of Rs. 50 million per year for 7 years. The company's cost of capital is 16 percent. What is the NPV and IRR of the differential project.

Solution:

Difference in capital outlays between projects P and Q is Rs.300 million
Difference in net annual cash flow between projects A and B is Rs.50 million.

$$\begin{aligned} \text{NPV of the differential project at 16 \%} \\ &= -300 + 50 \times \text{PVIFA (16 \%,7)} \\ &= -\text{Rs.98.05 million} \end{aligned}$$

IRR (r'') of the differential project can be obtained from the equation

$$50 \times \text{PVIFA}(r'', 7) = 300$$

By trial and error we get $r'' = 4.01\%$

10. Maharaja Associates is considering a project which requires an initial outlay of Rs.100 million. The cost of capital is 15 percent and the expected cash inflows from these projects are:

Year	1	2	3	4	5
Cash flow in Rs. Million	20	30	30	50	70

- (a) What is the payback period ?
(b) What is the discounted payback period ?
(c) What is the Benefit Cost Ratio ?

(a)

The pay back period of the project lies between 3 and 4 years. Interpolating in this range we get an approximate pay back period of $(3 + 20/50)$ or 3.4 years.

(b)

(Rs.in million)			
Year	Cash flow	PV of cash flow @15%	Cumulated PV
1	20	17.39	17.39
2	30	22.68	40.08
3	30	19.73	59.80
4	50	28.59	88.39
5	70	34.80	123.19

The discounted payback period = $4 + (100-88.39)/123.19 = 4.09$ years.

(c)

$$\begin{aligned} \text{PV of benefits (PVB)} &= 20 \times \text{PVIF (15\%,1)} + 30 \times \text{PVIF (15\%,2)} + 30 \times \text{PVIF (15\%,3)} \\ &\quad + 50 \times \text{PVIF (15\%,4)} + 70 \times \text{PVIF (15\%,5)} \\ &= 20 \times 0.870 + 30 \times 0.756 + 30 \times 0.658 + 50 \times 0.572 + 70 \times 0.497 \\ &= 123.21 \text{ million} \quad \text{(A)} \end{aligned}$$

$$\text{Investment} = 100 \text{ million} \quad \text{(B)}$$

$$\text{Benefit cost ratio} = 1.23 [= (A) / (B)]$$

11. The following financial information is available about a project:

(Rs. in million)

Year	1	2	3	4	5	6	7	8
Investment	3.60	3.15	2.70	2.25	1.80	1.35	0.90	0.45
Depreciation	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
Income before interest and taxes	0.90	0.95	0.80	0.85	0.90	0.70	0.85	0.80
Interest	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Income before tax	0.60	0.65	0.50	0.55	0.60	0.40	0.55	0.50
Tax	0.12	0.13	0.10	0.11	0.12	0.08	0.11	0.10
Income after tax	0.48	0.52	0.40	0.44	0.48	0.32	0.44	0.40

Compute the various measures of accounting rate of return.

Solution:

Year	1	2	3	4	5	6	7	8	Sum	Average
Investment	3.60	3.15	2.70	2.25	1.80	1.35	0.90	0.45	16.20	2.03
Depreciation	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	3.60	0.45
Income before interest and taxes	0.90	0.95	0.80	0.85	0.90	0.70	0.85	0.80	6.75	0.84
Interest	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	2.40	0.30
Income before tax	0.60	0.65	0.50	0.55	0.60	0.40	0.55	0.50	4.35	0.54
Tax	0.12	0.13	0.10	0.11	0.12	0.08	0.11	0.10	0.87	0.11
Income after tax	0.48	0.52	0.40	0.44	0.48	0.32	0.44	0.40	3.48	0.44

Measures of Accounting Rate of Return

- A.
$$\frac{\text{Average income after tax}}{\text{Initial investment}} = \frac{0.44}{3.6} = 12.22 \%$$
- B.
$$\frac{\text{Average income after tax}}{\text{Average investment}} = \frac{0.44}{2.03} = 21.67 \%$$
- C.
$$\frac{\text{Average income after tax but before interest}}{\text{Initial investment}} = \frac{0.44 + 0.3}{3.6} = 20.56 \%$$
- D.
$$\frac{\text{Average income after tax but before interest}}{\text{Average investment}} = \frac{0.44 + 0.3}{2.03} = 36.45 \%$$
- E.
$$\frac{\text{Average income before interest and taxes}}{\text{Initial investment}} = \frac{0.84}{3.6} = 23.33 \%$$
- F.
$$\frac{\text{Average income before interest and taxes}}{\text{Average investment}} = \frac{0.84}{2.03} = 41.38 \%$$
- G.
$$\frac{\text{Total income after tax but before Depreciation} - \text{Initial investment}}{(\text{Initial investment} / 2) \times \text{Years}} = \frac{3.48 + 3.60 - 3.60}{(3.6 / 2) \times 8} = 24.17 \%$$

CHAPTER 9

PROJECT CASH FLOWS

1. Gordhandas Ltd. is planning a project involving replacement of an old machine with a new machine. The old machine bought a few years ago has a book value of Rs. 12 lakhs and it can be sold to realise a post-tax salvage value of Rs. 15 lakhs. It has a remaining life of 4 years after which its net salvage value is expected to be Rs. 3 lakhs. It is being depreciated annually at a rate of 25 percent under WDV method.

The new machine costs Rs. 50 lakhs. It is expected to fetch a net salvage value of Rs. 24 lakhs after 4 years. The depreciation rate applicable to it is 25 percent under WDV method. The incremental working capital associated with this machine is Rs. 8 lakhs and it is expected to be recovered at its book value at the end of 4 years. The new machine is expected to bring a savings of Rs. 9 lakhs annually in manufacturing costs (other than depreciation). The tax rate applicable to the firm is 32 percent.

Estimate the cash flow associated with the replacement project.

Solution

A. Initial outlay (Time 0)

i. Cost of new machine	Rs. 50,00,000
ii. Salvage value of old machine	15,00,000
iii Incremental working capital requirement	800,000
iv. Total net investment (= i – ii + iii)	43,00,000

B. Operating cash flow (years 1 through 4)

Year	1	2	3	4
i. Post-tax savings in				
manufacturing costs	900,000	900,000	900,000	900,000
ii. Incremental depreciation	950,000	712,500	534,375	400,781
iii. Tax shield on incremental dep.	304,000	228,000	171,000	128,250
iv. Operating cash flow (i + iii)	12,04,000	11,28,000	10,71,000	10,28,250

Terminal cash flow:

i. Salvage value of new machine	Rs. 24,00,000
ii. Salvage value of old machine	300,000
iii. Recovery of incremental working capital	800,000
iv. Terminal cash flow (i – ii + iii)	29,00,000

D. Net cash flows associated with the replacement project (in Rs)

Year	0	1	2	3	4
NCF	(43,00,000)	12,04,000	11,28,000	10,71,000	39,28,250

2. Metaland have recently developed a prototype for a new light commercial vehicle labeled Meta 4 and you have been entrusted with the task of evaluating the project.

Meta 4 would be produced in the existing factory which has enough space for one more product. Meta 4 will require plant and machinery that will cost Rs.400 million. You can assume that the outlay on plant and machinery will be incurred over a period of one year. For the sake of simplicity assume that 50 percent will be incurred right in the beginning and the balance 50 percent will be incurred after 1 year. The plant will commence operation after one year.

Meta 4 project will require Rs.200 million toward gross working capital. You can assume that gross working capital investment will occur after 1 year.

The proposed scheme of financing is as follows: Rs.200 million of equity, Rs.200 million of term loan, Rs.100 million of working capital advance, and Rs.100

million of trade credit. Equity will come right in the beginning by way of retained earnings. Term loan and working capital advance will be raised at the end of year 1.

The term loan is repayable in 8 equal semi-annual instalments of Rs.25 million each. The first instalment will be due after 18 months of raising the term loan. The interest rate on the term loan will be 14 percent. The levels of working capital advance and trade credit will remain at Rs.100 million each, till they are paid back or retired at the end of 5 years, after the project commences, which is the expected life of the project. Working capital advance will carry an interest rate of 12 percent.

Meta 4 project is expected to generate a revenue of Rs.750 million per year. The operating costs (excluding depreciation and taxes) are expected to be Rs.525 million per year. For tax purposes, the depreciation rate on fixed assets will be 25 percent as per the written down value method. Assume that there is no other tax benefit.

The net salvage value of plant and machinery is expected to be Rs.100 million at the end of the project life. Recovery of working capital will be at book value. The income tax rate is expected to be 30 percent.

You are required to estimate the cash flows from three different points of view:

- a. Cash flows from the point of all investors (which is also called the explicit cost funds point of view).
- b. Cash flows from the point of equity investors.
- c. Cash flows as defined by financial institutions.

Cash Flows from the Point of all Investors

Item	0	1	2	3	4	5	6
1. Plant and equipment	(200)	(200)					
2. Net working capital		(100)					
3. Revenue			750	750	750	750	750
4. Operating costs			525	525	525	525	525
5. Depreciation			100	75	56.3	42.2	31.6
6. Profit before tax			125	150	168.7	182.8	193.4
7. Profit after tax (0.7 x 6)			87.5	105	118.1	128.0	135.4
8. Net salvage value of plant and equipment							100
9. Recovery of net working capital							100
10. Initial investment	(200)	(300)					
11. Operating cash flow (7 + 5)			187.5	180	174.4	170.2	167
12. Terminal cash inflow							200
13. Net cash flow	(200)	(300)	187.5	180	174.4	170.2	367

Cash Flows from the Point of Equity Investors

Item	0	1	2	3	4	5	6
1. Equity funds	(200)						
2. Revenues			750	750	750	750	750
3. Operating costs			525	525	525	525	525
4. Depreciation			100	75	56.3	42.2	31.6
5. Interest on working capital			12	12	12	12	12
6. Interest on term loan			28	26.3	19.3	12.3	5.3
7. Profit before tax			85	111.7	137.4	158.5	176.1
8. Profit after tax			59.5	78.2	96.2	111	123.3
9. Net salvage value of plant & equipment							100
10. Recovery of working capital							200
11. Repayment of term loans				50	50	50	50
12. Repayment of working capital advance							100
13. Retirement of trade credit							100
14. Initial investment (1)	(200)						
15. Operating cash inflows (8 + 4)			159.5	153.2	152.5	153.2	154.9
16. Liquidation & retirement cash flows (9 + 10 – 13 – 14 – 15)				(50)	(50)	(50)	50
17. Net cash flow	(200)	-	159.5	103.2	102.5	103.2	204.9

Cash Flows as defined by Financial Institutions

Item	0	1	2	3	4	5	6
1. Plant and equipment	(200)	(200)					
2. Net working capital		(100)					
3. Revenues			750	750	750	750	750
4. Operating costs			525	525	525	525	525
5. Depreciation			100	75	56.3	42.2	31.6
6. Interest on working capital			12	12	12	12	12
7. Interest on term loan			28	26.3	19.3	12.3	5.3
8. Profit before tax			85	111.8	137.5	158.6	176.1
9. Profit after tax			59.5	78.2	96.3	111	123.3
10. Recovery of net working capital							100
11. Residual value of capital assets							20
12. Initial investment (1 + 2)	(200)	(300)					
13. Operating inflow (8 + 4 + 5 + 6)			199.5	191.5	183.9	177.5	172.2
14. Terminal inflow (9 + 10)							120
15. Net cash flow (11 + 12 + 13)	(200)	(300)	199.5	191.5	183.9	177.5	192.2

3. Modern Foods is seriously considering a proposal for a lemon juice project. The lemon juice would be produced in an unused building adjacent to the main plant of Modern Foods. The building, owned by Modern Foods, is fully depreciated. However, it can be rented out for an annual rental of Rs.1 million. The outlay on the project is expected to be Rs.25 million - Rs.15 million toward plant and machinery and Rs.10 million toward gross working capital. You can assume that the outlay will occur right in the beginning. This means that there is no interest during the construction period.

The proposed scheme of financing is as follows: Rs.10 million of equity, Rs.8 million of term loan, Rs.5 million of working capital advance, and Rs.2 million of trade credit. The term loan is repayable in 8 equal semi-annual instalments of Rs.1 million each. The first instalment will be due after 18 months. The interest on the term loan will be 15 percent.

The levels of working capital advance and trade credit will remain at Rs.5 million and Rs.2 million respectively, till they are paid back or retired at the end of 5 years, which is the expected life of the project. Working capital advance will carry an interest rate of 14 percent. The lemon juice project is expected to generate a revenue of Rs.30 million a year. The operating costs (excluding depreciation and interest) are expected to be Rs.20 million a year.

For tax purposes, the depreciation rate on fixed assets will be 25 percent as per the written down value method. Assume that there is no other tax benefit. The net salvage value of plant and machinery is expected to be Rs.5 million at the end of year 5. Recovery of working capital, at the end of year 5, is expected to be at book value. The income tax rate is expected to be 30 percent.

Estimate the cash flows from the point of equity investors.

Solution:

Cash Flows from the Point of Equity Investors

Item	0	1	2	3	4	5
1. Equity funds	(10)					
2. Revenues		30	30	30	30	30
3. Costs (other than depreciation and interest)		20	20	20	20	20
4. Loss of rental		1	1	1	1	1
5. Depreciation		3.75	2.813	2.109	1.582	1.187
6. Interest on working capital advance		0.70	0.70	0.70	0.70	0.70
7. Interest on term loans		1.20	1.125	0.825	0.525	0.225
8. Profit before tax		3.35	4.362	5.366	6.193	6.888
9. Tax		1.005	1.309	1.610	1.858	2.066
10. Profit after tax		2.345	3.053	3.756	4.335	4.822
11. Net salvage value of fixed assets						5.000
12. Net salvage value of current assets						10.000
13. Repayment of term term loans			2.000	2.000	2.000	2.000
14. Repayment of bank advance						5.000
15. Retirement of trade creditors						2.000
16. Initial investment	(10)					
17. Operating cash inflow		6.095	5.866	5.865	5.917	6.009
18. Liquidation and retirement cash flows			(2.0)	(2.0)	(2.0)	6.00
19. Net cash flow	(10)	6.095	3.866	3.865	3.917	12.009

4. Rane Home Appliances Ltd is considering the manufacture of a new Dishwasher B-10, for which the following information has been gathered.

B-10 is expected to have a product life cycle of five years after which it will be withdrawn from the market. The sales from this product are expected to be as follows:

Year	1	2	3	4	5
Sales (Rs. in million)	800	950	1000	1200	1000

- The capital equipment required for manufacturing B-10 costs Rs. 900 million and it will be depreciated at the rate of 25 percent per year as per the WDV method for tax purposes. The expected net salvage value after 5 years is Rs. 150 million.
- The working capital requirement for the project is expected to be 10% of sales. Working capital level will be adjusted at the beginning of the year in relation to the sales for the year. At the end of five years, working capital is expected to be liquidated at par, barring an estimated loss of Rs. 5 million on account of bad debt, which of course, will be tax-deductible expense.
- The accountant of the firm has provided the following estimates for the cost of B-10

Raw material cost	:	45 percent of sales
Variable manufacturing cost	:	15 percent of sales
Fixed annual operating and maintenance costs	:	Rs. 3 million
Variable selling expenses	:	10 percent of sales
- The tax rate for the firm is 30 percent.
 - Estimate the post-tax incremental cash flows for the project to manufacture B-10.
 - What is the NPV of the project if the cost of capital is 20 percent?

Solution:

Cash flows for the B-10Project

(Rs. in million)

Year	0	1	2	3	4	5
1. Capital equipment	900					
2. Level of working capital	80	95	100	120	100	-
3. Revenues		800	950	1000	1200	1000
4. Raw material cost		360	427.5	450	540	450
5. Variable manufacturing cost		120	142.5	150	180	150
6. Operating and maintenance cost		3.0	3.0	3.0	3.0	3.0
7. Variable selling expenses		80	95	100	120	100
8. Depreciation		225	168.75	126.56	94.92	71.19
9. Bad debt loss						5
10. Profit before tax		12.0	113.25	170.44	262.08	220.81
11. Tax		3.6	33.98	51.13	78.62	66.24
12. Profit after tax		8.4	79.27	119.31	183.46	154.57
13. Net Salvage Value of Capital Equipment						150
14. Recovery of Working Capital						95
15. Initial Investment	(900)					
16. Operating cash flow (12+8+9)		233.40	248.02	245.87	278.38	230.76
17. Terminal cash flow (13 + 14)						245
18. Working Capital investment	(80)	(15)	(5)	(20)	20	
19. Net cash flow (15 + 16 + 17 + 18)	(980)	218.4	243.02	225.87	298.38	475.76

$$\begin{aligned}
 \text{(b) NPV} &= -980 + \frac{218.40}{(1.20)} + \frac{243.02}{(1.20)^2} + \frac{225.87}{(1.20)^3} + \frac{298.38}{(1.20)^4} + \frac{475.76}{(1.20)^5} \\
 &= -980 + 182 + 168.76 + 130.71 + 143.89 + 191.19 \\
 &= -\text{Rs. 163.45 million}
 \end{aligned}$$

CHAPTER 10

THE COST OF CAPITAL

1. A company issued 8 year, 12 percent bonds three years ago The bond which has a face value of Rs 1000 is currently selling for Rs 990.
 - a. What is the pre-tax cost of debt?
 - b. What is the after-tax cost of debt? (assume a 30 percent tax rate)

Solution:

- (a) Pre-tax cost of debt

$$= \frac{120 + (1000 - 990)/5}{0.4 \times 1000 + 0.6 \times 990} = 0.1227 \text{ or } 12.27 \%$$

- (b) After tax cost = $12.27 \times (1 - 0.30) = 8.59 \%$

2. Orient Corporation issued 15 year, 10 percent preference shares five years ago. The preference share which has a face value of Rs 100 is currently selling for Rs 105. What is the cost of preference shares?

Solution:

Using the approximate yield formula , cost of preference shares

$$= \frac{10 + (100 - 105)/10}{0.4 \times 100 + 0.6 \times 105} = 0.0922 \text{ i.e. } 9.22 \%$$

3. Nitin Corporation has a target capital structure of 70 percent equity and 30 percent debt. Its cost of equity is 15 percent and its pre-tax cost of debt is 12 percent. If the relevant tax rate is 32 percent, what is Nitin Corporation's WACC?

Solution:

$$\begin{aligned} \text{WACC} &= 0.3 \times 12 \% \times (1 - 0.32) + 0.7 \times 15 \% \\ &= 12.95 \% \end{aligned}$$

4. Omega Company's equity beta is 1.4. The risk-free rate is 8 percent and the market risk premium is 7 percent. Omega's debt-equity ratio is 0.8:1. Its pre-tax cost of debt is 12 percent. If the tax rate is 35 percent, what is its WACC ?

Solution:

$$r_E = 8 + 1.4 \times 7 = 17.8\%$$

$$\begin{aligned} r_D &= 12\% & T_c &= 0.35 \\ E/V &= 1 / 1.8 & D/V &= 0.8 / 1.8 \end{aligned}$$

$$WACC = \frac{E}{V} r_E + \frac{D}{V} r_D (1 - T_c)$$

$$WACC = \frac{1}{1.8} \times 17.8 + \frac{0.8}{1.8} \times 12 (1 - .35) = 13.36\%$$

5. Vinay Company's WACC is 10 percent and its tax rate is 35 percent. Vinay Company's pre-tax cost of debt is 10 percent and its debt-equity ratio is 1:1. The risk-free rate is 8 percent and the market risk premium is 7 percent. What is the beta of Vinay Company's equity ?

Solution:

$$\begin{aligned} WACC &= \frac{E}{V} r_E + \frac{D}{V} r_D (1 - T_c) \\ WACC &= 10\%, E/V = 0.5, D/V = 0.5, r_D = 10\%, T_c = 0.35 \\ 10\% &= 0.5 r_E + 0.5 \times 10\% \times (1 - .35) \\ r_E &= 13.50\% \\ r_E &= 8\% + \beta_E \times 7\% = 13.50\% \\ \beta_E &= 0.79 \end{aligned}$$

6. A company at present has total debt of Rs.100 million on its balance sheet and the interest payable thereon for the next year will be Rs.10 million. It wants to raise a further debt of Rs.20 million on the first day of the next year. What will be the total interest payable by it next year?

Solution:

The given data is insufficient to answer this question. We do not know the interest rate at which the company will be able to raise the further debt of Rs.20 million (i.e., we do not know its marginal cost of debt.). The interest payable by it on its existing debt is calculated based on its weighted average cost of debt, which is only a historical figure and is not necessarily equal to its marginal cost of debt.

7. V.R.Associates has 10 million equity shares outstanding. The book value per share is Rs 30 and the market price per share is Rs 100. V.R.Associates has two debenture issues outstanding. The first issue has a face value of Rs 200 million, 11 percent coupon, and sells for 105 percent of its face value. It will mature in 4 years. The second issue has a face value of Rs 300 million, 12 percent coupon, and sells for 92 percent of its face value. It will mature in 5 years. V.R.Associates also has a bank loan of Rs 100 million on which the interest rate is 14 percent.

What are V.R.Associates's capital structure weights on a book value basis and on a market value basis?

Solution:

The book value and market values of the different sources of finance are provided in the following table. The book value weights and the market value weights are provided within parenthesis in the table.

<i>(Rs. in million)</i>		
Source	Book value	Market value
Equity	300 (0.33)	1000 (0.63)
Debentures – first series	200 (0.22)	210 (0.13)
Debentures – second series	300 (0.33)	276 (0.17)
Bank loan	100 (0.11)	100 (0.06)
Total	900	1586

8. A company's capital structure in terms of market value is:

Debt	Rs 40 million
Equity	Rs 120 million

The company plans to maintain this market-value capital structure. The company has a plan to invest Rs 20 million next year. This will be financed as follows:

Retained earnings	Rs 2 million
Additional equity	Rs 10 million
Debt	Rs 4 million

The company's equity stock presently sells for Rs 50 per share. The next dividend expected is Rs 2.00. The expected rate of dividend growth is 10 percent. Additional equity can be issued at Rs 45 per share (net). The interest rate applicable to additional debt would be as follows:

First Rs 2 million	12 percent
Next Rs 2 million	14 percent

The tax rate for the firm is 32 percent.
Required:

- (a) At what amounts of new capital will there be breaks in the marginal cost of capital schedule?
- (b) What will be the marginal cost of capital in the interval between each of the breaks?

Solution:

$$\begin{aligned}\text{Cost of equity} &= D_1/P_0 + g \\ &= 2.00 / 50.00 + 0.10 \\ &= 14 \%\end{aligned}$$

- (a) The first chunk of financing will comprise of Rs.2 million of retained earnings costing 14 percent and Rs.0.67 million of debt costing $12(1-.32) = 8.16$ percent.

The second chunk of financing will comprise of Rs.10 million of additional equity costing 14 percent and Rs.3.33 million of debt costing $14(1-.32) = 9.52$ percent.

- (b) The marginal cost of capital in the first chunk will be:

$$0.75 \times 14\% + 0.25 \times 8.16 \% = 12.54 \%$$

The marginal cost of capital in the second chunk will be :

$$0.75 \times 14\% + 0.25 \times 9.52\% = 12.88 \%$$

Note : We have assumed that

- (i) The net realisation per share will be Rs.45, after floatation costs, and
- (ii) The planned investment of Rs. 20 million is inclusive of floatation costs

9. Hindustan Corporation has the following book value capital structure:

Equity capital (15 million shares, Rs 10 par)	Rs 150 million
Preference capital, 12 percent (200,000 shares, Rs 100 par)	Rs 20 million
Retained earnings	Rs 50 million
Debentures 14 percent (1,000,000 debentures, Rs 100 par)	Rs 100 million
Term loans, 15 percent	Rs 80 million
	Rs 400 million

The next expected dividend per share is Rs 3. The dividend per share is expected to grow at the rate of 10 percent. The market price per share is Rs 60.00. Preference stock, redeemable after 10 years, is currently selling for Rs 110 per share. Debentures, redeemable after 5 years, are selling for Rs 111 per debenture. The tax rate for the company is 30 percent.

- (a) Calculate the average cost of capital using
 - (i) book value proportions, and

- (ii) market value proportions
- (b) Define the marginal cost of capital schedule for the firm if it raises Rs 200 million next year, given the following information:
- (i) the amount will be raised from equity and debt in the ratio 2: 3
 - (ii) the firm expects to retain Rs 20 million earnings next year
 - (iii) the additional issue of equity stock will fetch a net price per share of Rs 56.00
 - (iv) the debt capital raised by way of term loans will cost 13 percent for the first Rs 100 million and 15 percent for the next Rs 20 million

Solution:

The cost of equity and retained earnings

$$r_E = \frac{D_1/P_0}{1} + g$$

$$= 3 / 60 + 0.10 = 15 \%$$

The cost of preference capital, using the approximate formula, is:

$$r_P = \frac{12 + (100-110)/10}{0.6 \times 110 + 0.4 \times 100} = 10.38 \%$$

The pre-tax cost of debentures, using the approximate formula, is:

$$r_D = \frac{14 + (100-111)/5}{0.6 \times 111 + 0.4 \times 100} = 11.07 \%$$

The post-tax cost of debentures is

$$11.07 (1 - 0.3) = 7.75 \%$$

The post-tax cost of term loans is

$$15 (1 - 0.3) = 10.5 \%$$

The average cost of capital using book value proportions is calculated below:

Source of capital	Component cost	Book value Rs. in million	Book value proportion	Product of (1) & (3)
	(1)	(2)	(3)	
Equity capital	15.0 %	150	0.375	5.63
Preference capital	10.38 %	20	0.05	0.52
Retained earnings	15.0 %	50	0.125	1.87
Debentures	7.75 %	100	0.25	1.94
Term loans	10.5 %	80	0.20	2.10
400 Average cost of capital				12.06 %

The average cost of capital using market value proportions is calculated below:

Source of capital	Component cost (1)	Market value Rs. in million (2)	Market value proportion (3)	Product of (1) & (3)
Equity capital and retained earnings	15.00 %	900	0.81	12.15
Preference capital	10.38 %	22	0.02	0.21
Debentures	7.75 %	111	0.10	0.78
Term loans	10.50 %	80	0.07	0.74
		1113	Average cost of capital=13.88 %	

(b)

The Rs.200 million to be raised will consist of the following:

Retained earnings Rs.20 million

Additional equity Rs. 60 million

Debt Rs. 120 million

The first batch will consist of Rs. 20 million of retained earnings and Rs. 30 million of debt costing 13 (1-0.3)= 9.1 percent respectively. The second batch will consist of Rs. 46.67 million of equity and Rs. 70 million of debt costing 13 (1-0.3)= 9.1 percent . The third chunk will consist of Rs.13.33 million of additional equity and 20 million of debt costing 14(1-0.3) = 9.8 percent..

The marginal cost of capital in the chunks will be as under

First batch: $(0.4 \times 15) + (0.6 \times 9.1) = 11.46 \%$

Second batch: $(0.4 \times 15) + (0.6 \times 9.1) = 11.46 \%$

Third batch : $(0.4 \times 15) + (0.6 \times 9.8) = 11.88 \%$

The marginal cost of capital schedule for the firm will be as under.

Range of total financing (Rs. in million)	Weighted marginal cost of capital (%)
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0 - 166.67	11.46
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166.67 - 200	11.88
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Here it is assumed that the Rs.200 million to be raised is inclusive of floatation costs.

10. Soumya Corporation is currently at its target debt-equity ratio of 1:1. It is considering a proposal to expand capacity which is expected to cost Rs 600 million

and generate after-tax cash flows of Rs 200 million per year for the next six years. The tax rate for the firm is 32 percent. The company has considered two financing options: (i) Issue of equity stock. The required return on the company's new equity is 25 percent and the issuance cost will be 10 percent. (ii) Issue of debentures at a yield of 14 percent. The issuance cost will be 2 percent.

- a. What is the WACC for Soumya Corporation?
- b. What is Soumya Corporation's weighted average floatation cost?
- c. What is the NPV of the proposal after taking into account the floatation costs?

Solution:

- (a) WACC = $0.5 \times 14\% \times (1 - 0.32) + 0.5 \times 25\%$
 = 17.26 %
- (b) Weighted average floatation cost
 = $0.5 \times 2\% + 0.5 \times 10\%$
 = 6 %
- (c) NPV of the proposal after taking into account the floatation costs
 = $200 \times \text{PVIFA}(17.26\%, 6) - 600 / (1 - 0.06)$
 = Rs.74.70 million

CHAPTER 11

PROJECT RISK ANALYSIS

1. A company has developed the following cash flow forecast for their new project.

	<i>Year 0</i>	<i>Rs. in million Years 1 - 10</i>
Investment	(400)	
Sales		440
Variable costs (75% of sales)		330
Fixed costs		20
Depreciation (Straight line method)		40
Pre-tax profit		50
Taxes (at 20 %)		10
Profit after taxes		40
Cash flow from operations		80
Net cash flow		80

What is the NPV of the new project? Assume that the cost of capital is 10 percent. The range of values that the underlying variables can take under three scenarios: pessimistic, expected and optimistic are as shown below:

<i>Underlying Variable</i>	<i>Pessimistic</i>	<i>Expected</i>	<i>Optimistic</i>
Investment (Rs. in million)	420	400	360
Sales (Rs. in million)	350	440	500
Variable cost as a percent of sales	80	75	70
Fixed costs (Rs. in million)	25	20	18
Cost of capital (%)	11	10	9

- (a) What are the NPVs under the different scenarios?
 (b) Calculate the accounting break-even point and the financial break-even point for the new project.
 Solution:

(a)

NPVs under alternative scenarios:

	<i>(Rs. in million)</i>		
	<i>Pessimistic</i>	<i>Expected</i>	<i>Optimistic</i>
Investment	420	400	360
Sales	350	440	500
Variable costs	280	330	350
Fixed costs	25	20	18

Depreciation	42	40	36
Pretax profit	3	50	96
Tax @ 20%	0.6	10	19.2
Profit after tax	2.4	40	76.8
Net cash flow	44.4	80	112.8
Cost of capital	11 %	10 %	9 %
NPV	- 158.53	91.6	363.95

Assumptions: (1) The useful life is assumed to be 10 years under all three scenarios. It is also assumed that the salvage value of the investment after ten years is zero.

(b) *Accounting break even point (under 'expected' scenario)*

Fixed costs + depreciation = Rs. 60 million
 Contribution margin ratio = $110/440 = 0.25$
 Break even level of sales = $60 / 0.25 = \text{Rs.}240 \text{ million}$

Financial break even point (under 'expected' scenario)

Annual net cash flow = $0.8[0.25 \times \text{sales} - 60] + 40$
 $= 0.2 \text{ sales} - 8$
 PV (net cash flows) = $[0.2 \text{ sales} - 8] \times \text{PVIFA}(10\%, 10)$
 $= [0.2 \text{ sales} - 8] \times 6.145$
 Initial investment = 400

At the financial break even level of sales
 $[0.2 \text{ sales} - 8] \times 6.145 = 400$
 or Sales = $(400/6.145 + 8) / 0.2 = 365.47 \text{ million.}$

2. Jawahar Industries has identified that the following factors, with their respective expected values, have a bearing on the NPV of their new project.

Initial investment	10,000
Cost of capital	11 %
Quantity manufactured and sold annually	1,000
Price per unit	20
Variable cost per unit	15
Fixed costs	1,000
Depreciation	1,000
Tax rate	20 %
Life of the project	7 years
Net salvage value	Nil

Assume that the following underlying variables can take the values as shown below:

<i>Underlying variable</i>	<i>Pessimistic</i>	<i>Optimistic</i>
Quantity manufactured and sold	700	1,400
Price per unit	18	23
Variable cost per unit	16	14

- (a) Calculate the sensitivity of net present value to variations in (a) quantity manufactured and sold, (b) price per unit, and (c) variable cost per unit.

- (a) Sensitivity of NPV with respect to quantity manufactured and sold:

	<i>Pessimistic</i>	<i>Expected</i>	<i>Optimistic</i>
Initial investment	10,000	10,000	10,000
Sale revenue	14,000	20,000	28,000
Variable costs	10,500	15,000	21,000
Fixed costs	1,000	1,000	1,000
Depreciation	1,000	1,000	1,000
Profit before tax	1,500	3,000	5,000
Tax	300	600	1,000
Profit after tax	1,200	2,400	4,000
Net cash flow	2,200	3,400	5,000
NPV at PVIFA(11%,7years) = 4.712	366	6,021	13,560

- (b) Sensitivity of NPV with respect to variations in unit price.

	<i>Pessimistic</i>	<i>Expected</i>	<i>Optimistic</i>
Initial investment	10,000	10,000	10,000
Sale revenue	18,000	20,000	23,000
Variable costs	15,000	15,000	15,000
Fixed costs	1,000	1,000	1,000
Depreciation	1,000	1,000	1,000
Profit before tax	1,000	3,000	6,000
Tax	200	600	1,200
Profit after tax	800	2,400	4,800
Net cash flow	1,800	3,400	5,800
NPV at PVIFA(11%,7years) = 4.712	- 1,518	6,021	17,330

- (c) Sensitivity of NPV with respect to variations in unit variable cost.

	<i>Pessimistic</i>	<i>Expected</i>	<i>Optimistic</i>
Initial investment	10,000	10,000	10,000
Sale revenue	20,000	20,000	20,000
Variable costs	16,000	15,000	14,000
Fixed costs	1,000	1,000	1,000
Depreciation	1,000	1,000	1,000
Profit before tax	2,000	3,000	4,000
Tax	400	600	800
Profit after tax	1,600	2,400	3,200
Net cash flow	2,600	3,400	4,200
NPV at PVIFA(11%,7years) = 4.712	2,251	6,021	9,790

3. A project involving an outlay of Rs.5 million has the following benefits associated with it.

<i>Year 1</i>		<i>Year 2</i>		<i>Year 3</i>	
<i>Cash Flow</i>	<i>Prob.</i>	<i>Cash Flow</i>	<i>Prob.</i>	<i>Cash Flow</i>	<i>Prob.</i>
<i>(Rs. in mln)</i>		<i>(Rs. in mln)</i>		<i>(Rs. in mln)</i>	
2	0.2	2	0.4	1	0.4
3	0.3	3	0.3	2	0.2
1	0.5	4	0.3	4	0.4

Assume that the cash flows are independent. Calculate the expected net present value and the standard deviation of net present value assuming that $i = 12$ percent.

Solution:

Let A_t be the random variable denoting net cash flow in year t .

$$\begin{aligned}\overline{A_1} &= 2 \times 0.2 + 3 \times 0.3 + 1 \times 0.5 \\ &= 1.8\end{aligned}$$

$$\begin{aligned}\overline{A_2} &= 2 \times 0.4 + 3 \times 0.3 + 4 \times 0.3 \\ &= 2.9\end{aligned}$$

$$\begin{aligned}\overline{A_3} &= 1 \times 0.4 + 2 \times 0.2 + 4 \times 0.4 \\ &= 2.4\end{aligned}$$

$$\begin{aligned}
 \text{NPV} &= 1.8 / 1.12 + 2.9 / (1.12)^2 + 2.4 / (1.12)^3 - 5 \\
 &= \text{Rs.0.63 million} \\
 \sigma_1^2 &= [(2-1.8)^2 \times 0.2 + (3-1.8)^2 \times 0.3 + (1-1.8)^2 \times 0.5] = 0.76 \\
 \sigma_2^2 &= [(2-2.9)^2 \times 0.4 + (3-2.9)^2 \times 0.3 + (4-2.9)^2 \times 0.3] = 0.69 \\
 \sigma_3^2 &= [(1-2.4)^2 \times 0.4 + (2-2.4)^2 \times 0.2 + (4-2.4)^2 \times 0.4] = 1.84 \\
 \sigma^2 (\text{NPV}) &= \frac{\sigma_1^2}{(1.12)^2} + \frac{\sigma_2^2}{(1.12)^4} + \frac{\sigma_3^2}{(1.12)^6} = \frac{0.76}{(1.12)^2} + \frac{0.69}{(1.12)^4} + \frac{1.84}{(1.12)^6} \\
 &= 1.98 \\
 \sigma (\text{NPV}) &= \text{Rs.1.41 million}
 \end{aligned}$$

4. A project has a current outlay of Rs.30,000. The expected value and standard deviation of cash flows are:

Year	Expected Value	Standard Deviation
1	Rs. 18,000	Rs. 7,000
2	20,000	4,000
3	20,000	6,000
4	10,000	2,000

The cash flows are perfectly correlated. Calculate the expected net present value and standard deviation of net present value of this investment, if the risk-free interest rate is 6 percent.

Solution:

$$\begin{aligned}
 &\text{Expected NPV} \\
 &= \sum_{t=1}^4 \frac{A_t}{(1.06)^t} - 30,000 \\
 &= \frac{18,000}{(1.06)} + \frac{20,000}{(1.06)^2} + \frac{20,000}{(1.06)^3} + \frac{10,000}{(1.06)^4} - 30,000 \\
 &= [18,000 \times 0.943 + 20,000 \times 0.890 + 20,000 \times 0.840 + 10,000 \times 0.792] - 30,000 \\
 &= 29,494
 \end{aligned}$$

Standard deviation of NPV

$$\sum_{t=1}^4 \frac{\sigma_t}{(1.06)^t}$$

$$\begin{aligned}
 &= 7,000/(1.06) + 4,000 / (1.06)^2 + 6,000 / (1.06)^3 + 2,000 / (1.06)^4 \\
 &= 7,000 \times 0.943 + 4,000 \times 0.890 + 6,000 \times 0.840 + 2,000 \times 0.792 \\
 &= 16,785
 \end{aligned}$$

5. The expected cash flows of a project are given below:

<i>Year</i>	<i>Cash Flow</i>
0	(50,000)
1	10,000
2	30,000
3	20,000
4	20,000
5	10,000

What is the net present value of the project under certainty equivalent method, if the risk-free rate of return is 8 percent and the certainty equivalent factor behaves as per the equation: $\alpha_t = 1 - 0.08t$

Solution:

Year	Cash Flow	Certainty Equivalent Factor: $\alpha_t = 1 - 0.08t$	Certainty Equivalent value	Discount Factor at 8%	Present Value
0	-50000	1	-50000	1	-50000
1	10000	0.92	9200	0.926	8519
2	30000	0.84	25200	0.857	21596
3	20000	0.76	15200	0.794	12069
4	20000	0.68	13600	0.735	9996
5	10000	0.6	6000	0.681	4086
				NPV =	6266

6. Cryonics Limited is planning to launch a new product, which can be introduced initially in Western India or in the entire country. If the product is introduced only in Western India, the investment outlay will be Rs.30 million. After two years, Cryonics can evaluate the project to determine whether it should cover the entire country. For such expansion, it will have to incur an additional investment of Rs.25 million. To introduce the product in the entire country right in the beginning would involve an outlay of Rs.50 million. The product, in any case, will have a life of 5 years, after which the plant will have a zero net salvage value.

If the product is introduced only in Western India, the demand would be high or low with probabilities of 0.8 and 0.2 respectively and annual cash inflows of Rs.10 million and Rs.6.25 million respectively.

If the product is introduced in the entire country right in the beginning the demand would be high or low with probabilities of 0.6 and 0.4 and annual cash inflows of Rs.20 million and Rs.12.5 million respectively.

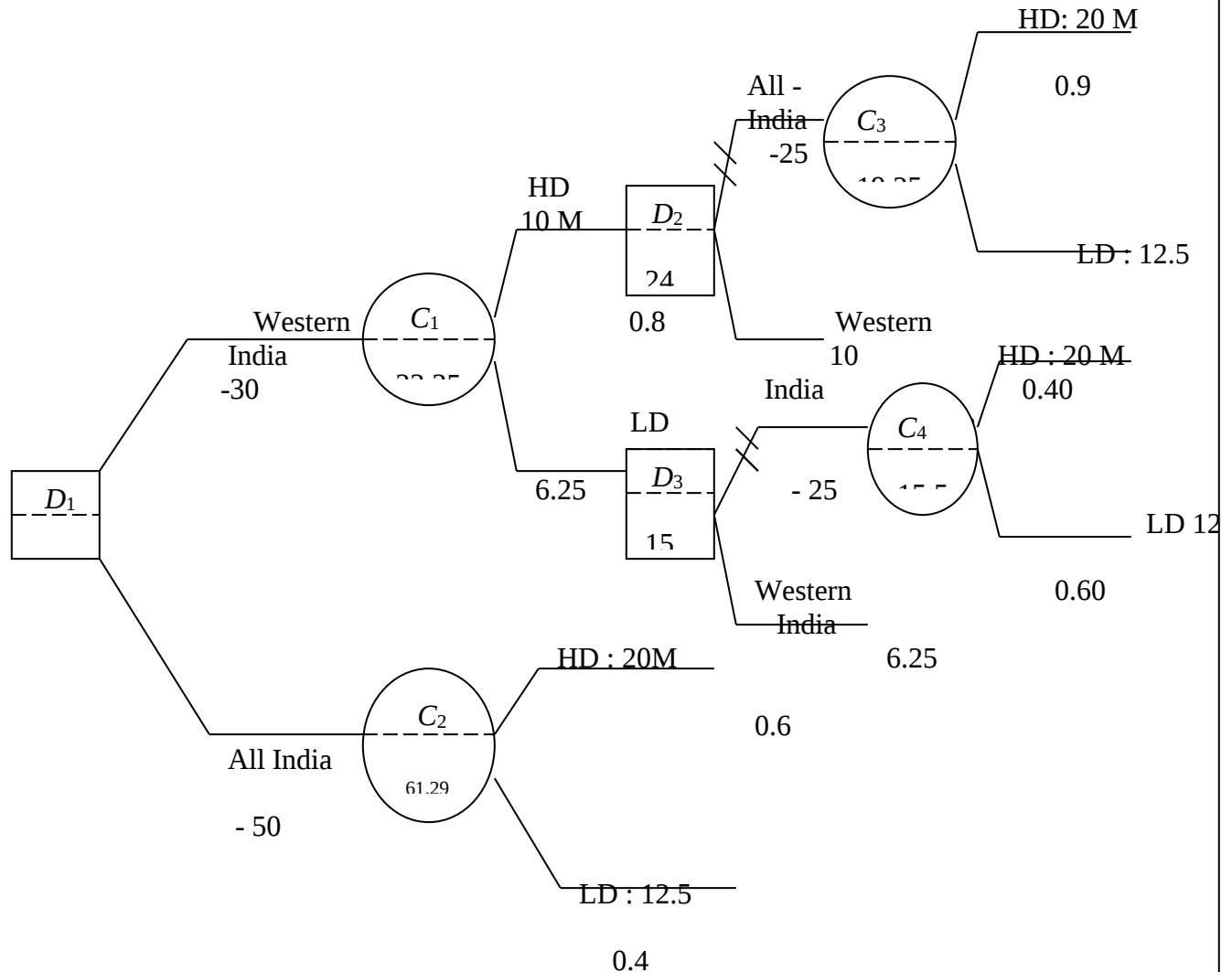
Based on the observed demand in Western India, if the product is introduced in the entire country the following probabilities would exist for high and low demand on an All-India basis.

Western India	All India	
	High demand	Low demand
High demand	0.90	0.10
Low demand	0.40	0.60

The hurdle rate applicable to the project is 12 percent.

- (a) Set up a decision tree for the investment situation of Cryonics Limited.

- (b) Advise Cryonics Limited on the investment policy it should follow. Support your advice with appropriate reasoning.



At D_2 the payoffs of the All India and Western India alternatives are:

All India : $19.25 \times \text{PVIFA}(3,12\%) - 25 = 21.2$ million

Western India : $10 \times \text{PVIFA}(3,12\%) = 24.0$ million

Since the Western India option is more profitable, the All-Indian option is truncated

At D_3 the payoffs of the All India and Western India alternatives are:

All India : $15.5 \times \text{PVIFA}(3,12\%) - 25 = 12.2$ million

Western India $6.25 \times \text{PVIFA}(3,12\%) = 15.0$

Since the Western India option is more profitable, the All-India option is truncated

At C_1 the expected payoff is:

$$\begin{aligned} & 0.8 [10 \times \text{PVIFA} (2,12\%) + 24 \times \text{PVIF} (2,12\%)] \\ & + 0.2 [6.25 \times \text{PVIFA} (2,12\%) + 15 \times \text{PVIF}(2,12\%)] \\ & = 0.8 [10 \times 1.736 + 24 \times 0.797] \\ & + 0.2 [6.25 \times 1.736 + 15 \times 0.797] \\ & = 0.8 [17.36 + 19.13] \\ & + 0.2 [10.85 + 11.96] = 29.19 + 4.56 = 33.75 \end{aligned}$$

At C_2 the expected payoff is :

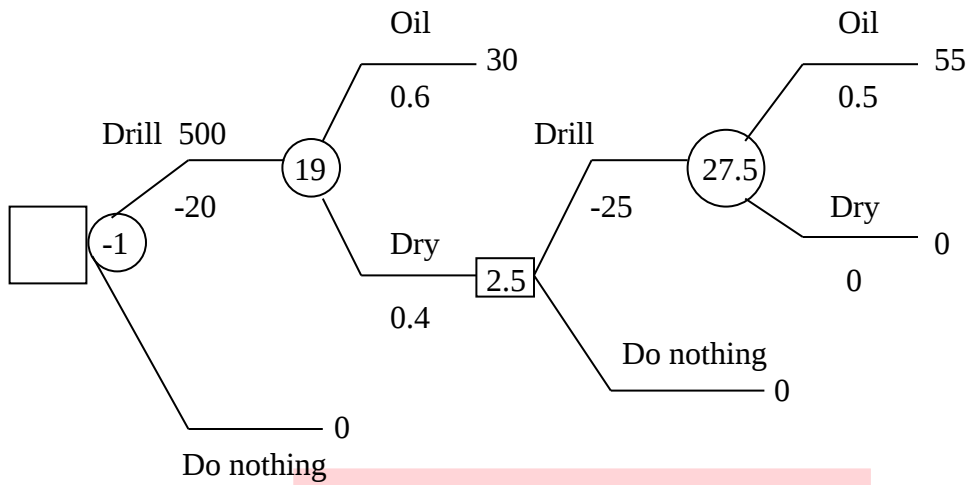
$$\begin{aligned} & [20 \times 0.6 + 12.5 \times 0.4] \times \text{PVIFA} (5,12\%) \\ & = 61.29 \end{aligned}$$

The appropriate investment policy is to choose the all-India alternative and continue with it.

7. Magna Oil is wondering whether to drill oil in a certain basin. The cost of drilling a 500 metre well is Rs.20 million. The probability of getting oil at that depth is 0.6. If oil is struck, the present value of oil obtained will be Rs.30 million. If the well turns out to be dry, Magna can drill another 500 metres at a cost of Rs.25 million. If it does so, the probability of striking oil at 1000 metres is 0.5 and the present value of oil obtained will be Rs.55 million.

Draw the decision tree. What is the optimal strategy for Magna Oil.

Working:



- Do nothing

CHAPTER 12

PROJECT RATE OF RETURN

- Vishal Enterprises is considering a shipping project for which it proposes to employ a debt-equity ratio of 2:1. Its pre-tax cost of debt will be 12 percent and its expected tax rate is 30 percent.

There are three firms, A, B, and C engaged wholly in shipping. Their tax rate is 35 percent. Their equity betas and debt-equity ratios are as follows:

	<i>Equity beta</i>	<i>Debt-equity ratio</i>
A	1.40	2.0
B	1.20	1.8
C	1.10	1.5

The risk-free rate is 8 percent and the expected return on the market portfolio is 14 percent.

- What is the average asset beta of the three firms A, B, and C. Solution:

<u>Firm</u>	<u>Asset Beta</u>	
	β_E	
	$\beta_A = \frac{1.4}{[1 + D/E (1 - T)]}$	
A :	$\frac{1.4}{1 + 2 (1 - .35)} = 0.609$	Average $(0.609 + 0.553 + 0.557)/3 = 0.573$
B :	$\frac{1.2}{1 + 1.8 (1 - .35)} = 0.553$	
C :	$\frac{1.10}{1 + 1.5 (1 - .35)} = 0.557$	

- What is the beta of the equity for the shipping project of Vishal Enterprises ?

Solution:

$$\begin{aligned}\beta_E &= \beta_A [1 + D/E (1 - T)] \\ &= 0.573 [1 + 2 (1 - .3)] = 1.375\end{aligned}$$

(iii) What is the required rate of return on the shipping project of Vishal Enterprises?

Solution:

$$r_E = 8\% + 1.375 (6\%) = 16.25\%$$

$$r_D = 12\%, T = 0.30$$

$$w_E = \frac{1}{3} \quad w_D = \frac{2}{3}$$

$$r_A = \frac{1}{3} \times 16.25 + \frac{2}{3} \times 12 (1 - .3) = 11.02\%$$

2. You have recently been appointed as the chief manager of the finance division of GNR Corporation. GNR is a diversified company with three independent divisions: Metals, Real Estate and Finance. The company evaluates the performance of each division based on a common cost of capital which is the cost of capital to the company. Being a finance person, you are unable to appreciate the logic of a common hurdle rate when the business profiles and risks involved of the three divisions are so different. When you take up the matter with the corporate office, the wise men there decide to use your finance expertise to the company's advantage. They ask you to work out separate costs of equity and hurdle rates for each division and send the report for approval of the board. They also ask you to list out the various measures that can be adopted to mitigate risk.

For the calculations you use the following details:

Exhibit 1 containing the latest balance sheet of the company with division- wise break up figures for assets and loans.

Exhibit 2 containing summarised financial statements and other details of three companies, Vajra Metals, Saheja Realty and Maxima Finance which are typical pure play companies with which the metals, real estate and finance divisions of GNR are respectively in active competition.

The risk-free rate currently is 9 percent and the general view is that the market risk premium is 10 percent. The corporate tax rate of 30 percent is applicable to all the businesses.

Exhibit 1 Balance Sheet of GNR Corporation					
• Shareholder's Funds		6000	• Fixed Assets		8200
• Capital	900		Metals :	2600	
• Reserves and surplus	5100		Real estate :	3800	
			Finance :	1800	
• Loan Funds		4000	• Net Current Asset		1800
• Term loan	2500		Metals :	800	

(12% interest)	:			Real estate	:	400	
Metals	:	400		Finance	:	600	
Real estate	:	1200					
Finance	:	900					
• Working capital loan (15% interest)		1500					
Metals	:	600					
Real estate	:	500					
Finance	:	400					
			10000				10000

Exhibit 2 Financial Data for the three companies			
	Vajra Metals	Saheja Realty	Maxima Finance
Fixed assets	4300	6800	2900
Net current assets	1300	800	1700
	5600	7600	4600
Share capital	1400	1300	1000
Reserves and Surplus	2600	3800	1200
Loan funds	1600	2500	2400
	5600	7600	4600
Revenues	7200	4000	3200
Net profit	800	500	300
Equity beta	1.4	0.9	0.80

You are required to answer the following questions.

- (1) What is the cost of equity applicable to the three divisions, viz, Metals, real estate, and finance?
- (2) What is the cut-off rate (cost of capital) applicable to the three divisions, viz, Metals, real estate, and finance?

Solution:

1.

a) Metals division

Asset beta of Vajra Metals : $\beta_A = \beta_E / [1 + (D/E) (1-T)]$

$$= 1.4 / [(1 + (1600/4000) \times (1 - 0.3))] = 1.094$$

By proxy this is the asset beta of the metals division also.

The equity beta of the metals division is therefore

$$\beta_E = \beta_A [1 + (D/E) (1-T)]$$

The total asset value of the metals division is 3400 out of which the debt component is 1000. So the equity component is $3400 - 1000 = 2400$

The debt-equity ratio for the electronics division is therefore $= 1000 / 2400$

$$\text{So } \beta_E = 1.094 [1 + (1000/2400) \times 0.3] = 1.231$$

Cost of equity = $R_f + \beta_E \times \text{Risk premium} = 9 + 1.231 \times 10 = 21.31\%$

Real Estate division

Asset beta of Saheja Realty = $0.9 / [1 + (2500/5100) \times 0.7] = 0.670$

By proxy this is the asset beta of the real estate division also.

The equity beta of the real estate division

$$\beta_E = 0.67 [1 + (1700/2500) \times 0.7] = 0.989$$

Cost of equity = $9 + 0.989 \times 10 = 18.89\%$

Finance division

Asset beta of Maxima Finance = $0.80 / [1 + (2400/2200) \times 0.7] = 0.454$

By proxy this is the asset beta of the finance division also.

The equity beta of the finance division

$$\beta_E = 0.454 [1 + (1300/1100) \times 0.7] = 0.830$$

Cost of equity = $9 + 0.830 \times 10 = 17.30\%$

2.

Metals division

$$\text{Post-tax weighted average cost of debt} = \left[\frac{400}{1000} \times 12 + \frac{600}{1000} \times 15 \right] (1 - 0.3) = 9.66\%$$

$$\text{Weighted average cost of capital} = \frac{1000}{3400} \times 9.66 + \frac{1400}{3400} \times 21.31 = 11.62\%$$

Real Estate division

$$\text{Post-tax weighted average cost of debt} = \left[\frac{1200}{1700} \times 12 + \frac{500}{1700} \times 15 \right] (1 - 0.3) = 9.02 \%$$

$$\text{Weighted average cost of capital} = \frac{2500}{4200} \times 18.89 + \frac{1700}{4200} \times 9.02 = 14.90 \%$$

Finance division

$$\text{Post-tax weighted average cost of debt} = \left[\frac{900}{1300} \times 12 + \frac{400}{1300} \times 15 \right] (1 - 0.3) = 9.05 \%$$

$$\text{Weighted average cost of capital} = \frac{1100}{2400} \times 17.30 + \frac{1300}{2400} \times 9.05 = 12.83 \%$$

CHAPTER 13

SPECIAL DECISIONS SITUATIONS

1. The contribution of a project which involves an outlay of 500 to the firm's debt capacity is 250. The project's opportunity cost of capital is 14 percent and the tax rate for the firm is 30 percent. The borrowing rate is 12 percent.

- (a) What is the adjusted cost of capital as per Modigliani and Miller formula?
- (b) What is the adjusted cost of capital as per Miles and Ezzell formula?

Solution:

Adjusted cost of capital as per Modigliani – Miller formula:

$$r^* = r(1 - TL)$$

$$r^* = 0.14(1 - 0.3 \times 0.5) = 0.119$$

Adjusted cost of capital as per Miles – Ezzell formula:

$$r^* = r - Lr_D T \frac{1 + r}{1 + r_D}$$

$$= 0.14 - 0.5 \times 0.12 \times 0.3 \times \frac{1 + 0.14}{1 + 0.12}$$

$$= 0.122$$

2. Sam Electricals is evaluating a capital project requiring an outlay of Rs.60 million. It is expected to generate a net cash inflow of Rs.15 million per year for 6 years. The opportunity cost of capital is 15 percent. Sam Electricals can raise a term loan of Rs.30 million for the project. The term loan will carry an interest rate of 12 percent. The principal amount would be repayable in 5 equal instalments, the first instalment falling due at the end of the second year. The balance amount required for the project can be raised by issuing external equity. The issue cost is expected to be 8 percent. The tax rate for the company is 30 percent.

- (i) What is the *base case* NPV?

Solution:

(In million)

- 60 + 15 x PVIFA (15 %, 6 years)
- 60 + 15 x 3.784 = - 3.24

- (ii) What is the *adjusted* NPV if the adjustment is made only for the issue cost of external equity?

$$\begin{aligned} \text{Solution: } & \frac{30,000,000}{0.92} = 32,608,696 \\ & \text{Issue cost} = 2.61 \text{ million.} \\ & \text{Adjusted NPV} = -3.24 - 2.61 = -5.85 \text{ million} \end{aligned}$$

- (iii) What is the present value of tax shield on debt finance?

Solution:				
		Rs. in million		
Year	Debt outstanding at beginning	Interest	Tax shield	PV at 12% discount rate
1	30	3.6	1.08	0.964
2	30	3.6	1.08	0.841
3	24	2.88	0.864	0.632
4	18	2.16	0.648	0.412
5	12	1.44	0.432	0.245
6	6	0.72	0.216	0.110
				<u>3.204</u>

- (iv) Now assume that the debt capacity of the project is 50% throughout. What is the adjusted cost of capital as per Miles and Ezzell formula?

$$\begin{aligned} \text{Solution: } & r^* = r - L r_D T \frac{1 + r}{1 + r_D} \\ & = 0.15 - 0.5 \times 0.12 \times 0.3 \times \frac{1.15}{1.12} = 13.15\% \end{aligned}$$

3. Overseas Ventures, an Indian company, is considering a project to be set up in the US. The project will entail an initial outlay of \$300 million and is expected to generate the following cash flow over its five year life:

Year	1	2	3	4	5

Cash flow (in million)	\$60	\$100	\$120	\$120	\$100
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The current spot exchange rate is Rs 47 per US dollar, the risk-free rate in India is 8 percent and the risk-free rate in the US is 2 percent.

Overseas Ventures's required rupee return on a project of this kind is 15 percent. Calculate the NPV of the project using the home currency approach.

Solution:

- . $S_0 = \text{Rs.}47$, $r_h = 8 \text{ per cent}$, $r_f = 2 \text{ per cent}$
Hence the forecasted spot rates are:

Year	Forecasted spot exchange rate
1	$\text{Rs.}47 (1.08 / 1.02)^1 = \text{Rs.}49.76$
2	$\text{Rs.} 47 (1.08 / 1.02)^2 = \text{Rs.}52.69$
3	$\text{Rs.} 47 (1.08 / 1.02)^3 = \text{Rs.}55.79$
4	$\text{Rs.} 47 (1.08 / 1.02)^4 = \text{Rs.}59.07$
5	$\text{Rs.} 47 (1.08 / 1.02)^5 = \text{Rs.}62.55$

The expected rupee cash flows for the project

Year	Cash flow in dollars (million)	Expected exchange rate	Cash flow in rupees (million)
0	-300	47	-14,100
1	60	49.76	2,986
2	100	52.69	5,269
3	120	55.79	6,695
4	120	59.07	7,088
5	100	62.55	6,255

Given a rupee discount rate of 20 per cent, the NPV in rupees is :

$$\begin{aligned}
 \text{NPV} &= -14,100 + \frac{2,986}{(1.15)} + \frac{5,269}{(1.15)^2} + \frac{6,695}{(1.15)^3} \\
 &\quad + \frac{7,088}{(1.15)^4} + \frac{6,255}{(1.15)^5} \\
 &= \text{Rs.}4045 \text{ million}
 \end{aligned}$$

The dollar NPV is:

$$4045 / 47 = 86 \text{ million dollars}$$

CHAPTER 15

MULTIPLE PROJECTS AND CONSTRAINTS

1. Five projects, *A*, *B*, *C*, *D*, and *E* are available to a company.

	<i>M</i>	<i>N</i>	<i>O</i>	<i>P</i>	<i>Q</i>
Initial investment	Rs 20,000	50,000	85,000	90,000	150,000
Annual cash inflow	Rs 5,000	10,000	20,000	20,000	25,000
Life (in years)	8	9	6	6	10
Salvage value	Rs 6,000	—	—	20,000	40,000

Projects *N* and *Q* are mutually exclusive. Otherwise the projects are independent. If the cost of capital for the firm is 12 percent, which projects should be chosen at the following budget levels: Rs 300,000 and Rs 350,000. Assume that the decision criterion is the net present value. Use the feasible combinations approach.

Solution:

The NPVs of the projects are as follows:

$$\text{NPV (M)} = 5,000 \times \text{PVIFA}(12\%,8) + 6,000 \times \text{PVIF}(12\%,8) - 20,000 = \text{Rs.}7,264$$

$$\text{NPV (N)} = 10,000 \times \text{PVIFA}(12\%,9) - 50,000 = \text{Rs.}3,280$$

$$\text{NPV (O)} = 20,000 \times \text{PVIFA}(12\%,6) - 85,000 = - \text{Rs.}2,780$$

$$\text{NPV(P)} = 20,000 \times \text{PVIFA}(12\%,6) + 20,000 \times \text{PVIF}(12\%,6) - 90,000 = \text{Rs.}2,360$$

$$\begin{aligned} \text{NPV (Q)} &= 25,000 \times \text{PVIFA}(12\%,10) + 40,000 \times \text{PVIF}(12\%,10) \\ &\quad - 150,000 = \text{Rs.}4,130 \end{aligned}$$

As the NPV of *O* is negative it is rejected. *N* and *Q* are mutually exclusive. The feasible combinations, their outlays, and their NPVs are given below.

<i>Combination</i>	<i>Outlay (Rs.)</i>	<i>NPV (Rs.)</i>
M	20,000	7,264
N	50,000	3,280
P	90,000	2,360
Q	150,000	4,130
M & N	70,000	10,544
M & P	110,000	9,624
M & Q	170,000	11,394
N & P	140,000	5,640
P & Q	240,000	6,490
M & N & P	160,000	12,904

The preferred combination is M & N & P

2. A firm is evaluating six investment opportunities:

Project (j)	Net present value (NPV _j)	Cash outflow in period 1 (CF _{j1})	Cash outflow in period 2 (CF _{j2})
	Rs	Rs	Rs
1	8,000	9,000	8,000
2	10,000	10,000	12,000
3	15,000	11,000	20,000
4	20,000	25,000	30,000
5	40,000	50,000	40,000
6	80,000	70,000	60,000

The budget available is limited to Rs 130,000 in year 1 and Rs 150,000 in year 2. Any amount not spent in year 1 can be transferred to year 2. The amount so transferred will earn a post-tax return of 6 percent.

There are two additional constraints: power constraint and managerial constraint. The requirements and constraints applicable in this respect are:

Project (j)	Power requirement (W _j)	Managerial requirement (M _j)
1	3,000	10
2	5,000	15
3	4,000	20
4	8,000	25
5	10,000	30
6	20,000	40
	$\sum X_j W_j \leq 50,000$	$\sum X_j M_j \leq 100$

Develop a linear programming formulation of the above capital budgeting problem.

Solution:

The linear programming formulation of the capital budgeting problem under various constraints is as follows:

$$\text{Maximise } 8 X_1 + 10 X_2 + 15 X_3 + 20 X_4 + 40 X_5 + 80 X_6$$

Subject to

$$9 X_1 + 10 X_2 + 11 X_3 + 25 X_4 + 50 X_5 + 70 X_6 + SF_1 = 130 \quad \text{Funds constraint for year 1}$$

$$8 X_1 + 12 X_2 + 20 X_3 + 30 X_4 + 40 X_5 + 60 X_6 \leq 150 + 1.06 SF_1 \quad \text{Funds constraint for year 2}$$

$$3 X_1 + 5 X_2 + 4 X_3 + 8 X_4 + 10 X_5$$

$$+ 20 X_6 \leq 50$$

Power constraint

$$10 X_1 + 15 X_2 + 20 X_3 + 25 X_4 + 30 X_5 \\ + 40 X_6 \leq 100$$

Managerial constraint

$$0 \leq X_j \leq 1 \ (j = 1, \dots, 8) \text{ and } SF_1 \geq 0$$

Rupees are expressed in '000s. Power units are also expressed in '000s.



CHAPTER 16

VALUATION OF REAL OPTIONS

1. A stock is currently selling for Rs.80. In a year's time it can rise by 50 percent or fall by 20 percent. The exercise price of a call option is Rs.90.
 - (i) What is the value of the call option if the risk-free rate is 10 percent? Use the option-equivalent method.

Solution: $S_0 = \text{Rs.}80$ $u = 1.5$ $d = 0.8$
 $E = \text{Rs.}90$ $r = 0.10$ $R = 1.10$

$$\Delta = \frac{C_u - C_d}{(u - d) S} = \frac{30 - 0}{0.7 \times 80} = \frac{30}{56}$$

$$B = \frac{u C_d - d C_u}{(u - d) R} = \frac{1.5 \times 0 - 0.8 \times 30}{0.7 \times 1.10} = -31.17$$

$$\begin{aligned} C &= \Delta S + B \\ &= \frac{30}{56} \times 80 - 31.17 \\ &= 11.69 \end{aligned}$$

- (ii) What is the value of the call option if the risk-free rate is 6 percent? Use the risk-neutral method.

Solution:

$$[P \times 50\%] + [(1 - P) \times -20\%] = 6\%$$

$$50P + 20P = 26 \Rightarrow P = 0.37$$

Expected future value of a call

$$0.37 \times 30 + 0.63 \times 0 = \text{Rs.}11.10$$

$$\text{Rs.}11.10$$

$$\text{Current value} = \frac{\text{Rs.}11.10}{1.06} = \text{Rs.}10.47$$

2. A company's equity is currently selling for Rs 150. In a year from now it can rise or fall. On the downside it may fall to Rs 120. The call option on Beta's equity has a value of Rs 18. If the interest rate is 8 percent, to what level would the company's equity rise on the upside? Assume that the exercise price is Rs 140.

Solution:

$$S = 150, dS = 120, d = 0.8, C = 18$$

$$r = 0.08, R = 1.08, E = 140$$

$$C_u = \text{Max} (uS - E, 0) = \text{Max} (150u - E, 0) = 150u - 140$$

$$C_d = \text{Max} (dS - E, 0) = \text{Max} (120 - 140, 0) = 0$$

$$\Delta = \frac{C_u - C_d}{(u-d)S} = \frac{150u - 140}{(u - 0.8)150}$$

$$B = \frac{u C_d - d C_u}{(u-d)R} = \frac{-0.8 (150u - 140)}{(u - 0.8) 1.08}$$

$$C = \Delta S + B$$

$$18 = \frac{(150u - 140) \times 150}{(u - 0.8) \times 150} - \frac{0.8 \times (150u - 140)}{1.08 \times (u - 0.8)}$$

Multiplying both the sides by $1.08(u - 0.8)$, we get

$$18 \times 1.08 \times (u - 0.8) = 1.08 \times (150u - 140) - 0.8 \times (150u - 140)$$

$$19.44u - 15.55 = 0.28(150u - 140) = 42u - 39.2$$

$$u = 1.048$$

So the company's equity stock can rise to

$$150 \times 1.048 = \text{Rs.}157$$

3. Consider the following data for a certain stock:

Price of the stock now = $S_0 = \text{Rs.}150$

Exercise price = $E = \text{Rs.}140$

Standard deviation of continuously compounded annual return = $\sigma = 0.30$

Expiration period of the call option = 3 months

Risk-free interest rate per annum = 6 percent

- (i) What is the value of the call option as per the Black-Scholes model? Use the normal distribution table and resort to linear interpolation.

Working: $C_0 = S_0 N(d_1) - \frac{E}{e^{rt}} N(d_2)$ $S_0 = \text{Rs.}150, E = \text{Rs.}140, r = 0.06,$

$d_1 = \frac{\ln(S_0/E) + (r + \sigma^2/2) t}{\sigma \sqrt{t}}$ $\sigma = 0.3, t = 0.25$

$$= \frac{0.069 + (0.06 + 0.09/2) 0.25}{0.3 \sqrt{0.25}} = 0.635$$

$d_2 = d_1 - \sigma \sqrt{t} = 0.485$

$N(d_1) = N(0.635) = 0.7373$

$N(d_2) = N(0.485) = 0.6861$

$$C_0 = 150 \times 0.7373 - \frac{140}{e^{0.06 \times 0.25}} \times 0.6861$$

$$= 110.60 - 94.62 = \text{Rs.}15.98$$

$N(0.60) = 1 - 0.2743 = 0.7257$

$N(0.65) = 1 - 0.2578 = 0.7422$

$N(0.635) = 0.7257 + \frac{.035}{.05} (.7422 - .7257)$

$= 0.7373$

$N(0.45) = 1 - 0.3264 = 0.6736$

$N(0.50) = 1 - 0.3085 = 0.6915$

$N(0.485) = 0.6736 + \frac{.035}{.05} (.6915 - 0.6736)$

$= 0.6861$

(ii) What is the value of the put option?

Solution:

$$P_0 = C_0 - S_0 + \frac{E}{e^{rt}}$$

$$= 15.98 - 150 + \frac{140}{e^{0.06 \times .25}}$$

$= \text{Rs.}3.90$

	0	1	2	3	4
• Initial outlay	(550)				
• After – tax operating cash flow		120	240	240	120
• Terminal cash flow					50

The discount rate applicable to Harmonica-I is 18 percent.

If the firm undertakes Harmonica-I proposal, it will be in a position to make a follow on investment in an advanced version, Harmonica-II, four years from now. Harmonica-II will be double the size of Harmonica-I in terms of investment outlay and cash inflows. The cash inflows of Harmonica-II would have a standard deviation of 30 percent per year. The risk-free interest rate is 10 percent.

- What is the net present value of Harmonica-I?
- What is the value of the option to invest in Harmonica –II?

Solution:

- The present value of the cash inflows of Harmonica – I is:

$$\frac{120}{(1.18)} + \frac{240}{(1.18)^2} + \frac{240}{(1.18)^3} + \frac{170}{(1.18)^4}$$

$$120 (0.847) + 240 (0.718) + 240 (0.609) + 170 (0.516) = 507.8$$

$$\text{Investment outlay} = 550$$

$$\text{NPV} = -550 + 507.8 = 42.2$$

- b. To value the option to invest in Harmonica – II we have to cast the information given in the case in terms of the inputs required by the Black – Scholes formula.

$$S_0 = \text{present value of the asset} = 507.8 \times e^{-0.18 \times 4} = 247.2$$

$$E = \text{exercise price} = 1100$$

$$\sigma = \text{standard deviation of the continuously compounded annual returns} = 0.3$$

$$t = \text{years of maturity} = 4$$

$$r = \text{interest rate per annum} = 12 \text{ percent}$$

Given the above inputs, the value of the option to invest in Harmonica – II may be calculated as follows:

Step 1 : calculate d_1 and d_2

$$d_1 = \frac{\ln \left(\frac{S_0}{E} \right) + \left(r + \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}} = \frac{-1.492 \left(+0.12 + \frac{0.09}{2} \right) 4}{\sigma \sqrt{4}}$$

$$= -1.3867$$

$$d_2 = -1.3867 - 0.3\sqrt{4} = -1.9867$$

Step 2: Find $N(d_1)$ and $N(d_2)$

$$N(-1.3867) = 0.808 + \frac{1.40 - 1.3867}{0.05} \times (0.0885 - 0.0808)$$

$$= 0.0828$$

$$N(-1.9867) = 0.0228 + \frac{2.00 - 1.9867}{0.05} (0.0256 - 0.0228)$$

$$= 0.0235$$

Step 3 Estimate the present value of the exercise price.

$$E \cdot e^{-rt} = 1100 / 2.0544 = \text{Rs. } 535.44$$

Step 4 Plug the numbers obtained in the previous steps in the Black – Scholes formula

$$C_0 = 247.2 \times 0.0828 - 535.44 \times 0.0235$$

$$= 20.47 - 12.58 = \text{Rs. } 7.89 \text{ million}$$

5. A builder owns a plot of land that can be used for either 72 or 120 apartment building. The construction costs of these alternatives are Rs.72 million and Rs.136 million respectively. The price per apartment currently is Rs.1.2 million. The yearly rental (net of expense) per unit is Rs.0.10 million and the risk-free rate of interest is 9 percent per annum.

If the market for the apartments is buoyant next year, each apartment will fetch Rs.1.5 million. On the other hand, if the market for the apartments is sluggish next year, each apartment will fetch Rs.1.1 million. Assume that the construction costs will remain unchanged.

Solution:

No. of apartments	Profit
72	$72(1.2) - 72 = 14.4 \text{ million}$
120	$120(1.2) - 136 = 8 \text{ million}$

- (ii) If the builder waits for one year what is the payoff from the best alternative if the market turns out to be buoyant.

Solution:

<u>Alternative</u>	<u>Buoyant Market</u> Apartment Price: Rs. 1.5 million
72 apartments	$72 \times 1.5 - 72 = 36$
120 apartments	$120 \times 1.5 - 136 = 44$

- (iii) If the builder waits for one year, what is the payoff from the best alternative, if the market turns out to be sluggish ?

Solution:

<u>Alternative</u>	<u>Sluggish Market</u> Apartment Price : 1.1 million
72 apartments	$72 \times 1.1 - 72 = 7.2$
120 apartments	$120 \times 1.1 - 136 = -4$

- (iv) What are the risk-neutral probabilities that the market for apartments will be buoyant and sluggish respectively?

Solution:

$$\begin{array}{lcl}
 & \begin{array}{c} 1.5 \\ + 0.1 = 1.6 \end{array} & \\
 & \begin{array}{c} b \\ \swarrow \end{array} & \\
 1.2 & & \\
 & \begin{array}{c} \searrow \\ 1-p \end{array} & \\
 & \begin{array}{c} 1.1 + 0.1 = 1.2 \end{array} &
 \end{array}
 \qquad
 \begin{array}{l}
 \frac{1.6p + (1-p) \times 1.2}{1.09} = 1.2 \\
 \\
 \text{This leads to} \\
 p = 0.27 \\
 \text{So } 1-p = 0.73
 \end{array}$$

- (v) What is the expected payoff next year?

Solution:

$$\begin{aligned}
 & 0.27 \times 44 + 0.73 \times 7.2 \\
 & = 11.88 + 5.256 \\
 & = \text{Rs. } 17.136 \text{ million}
 \end{aligned}$$

- (vi) What is the value of the vacant land?

Solution:

$$\frac{17.136}{1.09} = 15.72$$

6. Oriental Limited is assessing the value of the option to extract oil from a particular oil basin. The following information has been gathered:
- The estimated oil reserve in the basin is 200 million barrels of oil. Assume that there is no variability characterising this quantity.
 - The development cost is \$1000 million.
 - The right to exploit the basin will be enjoyed for 25 years.
 - The marginal value per barrel of oil presently is \$30—this represents the difference between the price per barrel of oil and the marginal cost of extracting a barrel of oil. The standard deviation of l_n (oil price) is estimated to be 0.2
 - Once developed, the net production revenue each year will be 8% of the value of the reserve.
 - The risk-free rate is 6 %.
 - The development lag is four years.
- What is the value of the option to extract oil?

Solution:

S_0 = current value of the asset = value of the developed reserve discounted for 4 years (the development lag) at the dividend yield of 8% = $\$30 \times 200 / (1.08)^4 = \$ 4410.2$ million.

E = exercise price = development cost = \$1000 million

σ = standard deviation of l_n (oil price) = 0.2

t = life of the option = 25 years

r = risk-free rate = 6 %

y = dividend yield = net production revenue/ value of reserve = 8 %

Given these inputs, the call option is valued as follows:

Step 1 :

Calculate d_1 and d_2

$$d_1 = \frac{l_n \left(\frac{S}{E} \right) + \left(r - y + \frac{\sigma^2}{2} \right) t}{\sigma \sqrt{t}}$$

$$\begin{aligned} &= \frac{l_n (4410.2 / 1000) + [.06 - .08 + (.04 / 2)] 25}{0.2 \sqrt{25}} = \\ &= \frac{1.4839}{1} = 1.4839 \end{aligned}$$

$$d_2 = d_1 - \sigma \sqrt{t} = 1.4839 - 1 = 0.4839$$

Step 2 : Using tables find $N(d_1)$ and $N(d_2)$ by intrapolation:

$$N(d_1) = N(1.4839) = [1 - (0.0668 + (1.50 - 1.4839) \times (0.0735 - 0.0668) / 0.05)] \\ = 0.9310$$

$$N(d_2) = N(0.4839) = [1 - (0.3085 + (0.50 - 0.4839) \times (0.3264 - 0.3085) / 0.05)] \\ = 0.6857$$

Step 3 : Estimate the present value of the exercise price

$$E / e^{rt} = 1000 / e^{0.06 \times 25} = 1000 / 4.4817 = \$ 223.13 \text{ million}$$

Step 4 : Plug the numbers obtained in the previous steps in the Black-Scholes formula:

$$C = \$4410.2 \text{ million} \times 0.9310 - \$ 223.13 \text{ million} \times 0.6857 \\ = \$ 3952.9 \text{ million}$$



CHAPTER 21

PROJECT MANAGEMENT

1. A project has begun on 1st July 200X and is expected to be completed by 31st December 200X. The project is being reviewed on 30th September 200X when the following information has been developed:

- Budgeted cost for work scheduled (BCWS) :Rs 8,000,000
- Budgeted cost for work performed (BCWP) :Rs 4,600,000
- Actual cost of work performed (ACWP) : Rs 4,100,000
- Budgeted cost for total work (BCTW) : Rs 11,000,000
- Additional cost for completion (ACC) : Rs 6,000,000

Determine the following: (i) cost variance, (ii) schedule variance in cost terms, (iii) cost

Solution:

i. Cost variance: $BCWP - ACWP = 4,600,000 - 4,100,000$
 $= \text{Rs. } 500,000$

ii. Schedule variance in cost terms: $BCWP - BCWS = 4,600,000 - 8,000,000$
 $= - \text{Rs. } 3,400,000$

iii. Cost performance index: $BCWP / ACWP = \frac{4,600,000}{4,100,000} = 1.12$

iv. Schedule performance index: $BCWP / BCWS = \frac{4,600,000}{8,000,000} = 0.575$

v. Estimated cost performance index: $\frac{BCTW}{(ACWP + ACC)} = \frac{11,000,000}{4,100,000 + 6,000,000}$
 $= 1.089$

CHAPTER 23

PROJECT REVIEW AND ADMINISTRATIVE ASPECTS

- Sibal Associates is considering a project involving an outlay of Rs 200 million. The projected cash inflows of this project over its 6-year life are as follows:

Year	1	2	3	4	5	6
Cash inflow	30	50	80	71	48	44

(Rs in million)

This investment is a zero-NPV investment at a discount rate of 15 percent. Calculate the economic rate of return and the book return on income (assuming a straight line depreciation over the 6-year life) for the above project.

Solution:

Calculation of Economic Rate of Return

	Year	1	2	3	4	5	6
1	Cash flow	30	50	80	71	48	44
2	Present value at the beginning of the year; 15 percent discount rate	199.977	199.973	179.969	126.965	75.009	38.261
3	Present value at the end of the year, 15 percent discount rate	199.973	179.969	126.965	75.009	38.261	0.000
4	Change in value during the year (3 – 2)	-0.003	-20.004	-53.005	-51.955	-36.749	-38.261
5	Economic income (1+4)	29.997	29.996	26.995	19.045	11.251	5.739
6	Economic rate of return (5/2)	0.15	0.15	0.15	0.15	0.15	0.15
7	Economic depreciation	0.003	20.004	53.005	51.955	36.749	38.261

Calculation of Book Return on Investment

	Year	1	2	3	4	5	6
1	Cash flow	30	50	80	71	48	44
2	Book value at the beginning of the year, straight line depreciation	200	166.667	133.33	100	66.667	33.333
3	Book value at the end of the year, straight line depreciation	166.667	133.33	100	66.667	33.333	0
4	Change in book value during the year (3 – 2)	-33.333	-33.337	-33.33	-33.333	-33.334	-33.333
5	Book income (1 + 4)	-3.333	16.663	46.67	37.667	14.666	10.667
6	Book return on investment (5/2)	-0.017	0.100	0.350	0.377	0.220	0.320
7	Book depreciation	33.333	33.337	33.33	33.333	33.334	33.333

2. Vijay Corporation had set up a project which has a remaining life of 5 years. The cash flow forecast for the balance life is as follows:

Year	1	2	3	4	5
Cash flow forecast	20	30	40	30	10

(Rs in million)

The salvage value of the project if terminated immediately is Rs 50 million. A third party has offered to buy the project for Rs 75 million. The discount rate is 15 percent. What should Vijay do?

Solution:

$SV = \text{Rs. } 50 \text{ million}$

$DV = \text{Rs. } 75 \text{ million}$

$$PVCF = \frac{20}{(1.15)} + \frac{30}{(1.15)^2} + \frac{40}{(1.15)^3} + \frac{30}{(1.15)^4} + \frac{10}{(1.15)^5} = 88.50 \text{ million}$$

Since $PVCF > DV > SV$ it is advisable to continue the project through its remaining life.